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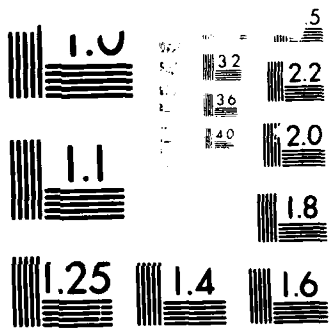
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EVALUATION OF A FREQUENCY RESPONSE TECHNIQUE

FOR AIRCRAFT SYSTEM IDENTIFICATION

THESIS

AFIT/GAE/AA/85J-2 ALLAN T. REED
CAPT USAF

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19. ABSTRACT (Continue on reverse if necessary and identify by block number)

This paper presents the results of a project which used frequency analysis methods applied to flight test data in order to identify aircraft parameters. Computer programs were developed to generate simulated flight test data so the frequency response programs could be tested using a noise free data source. Once the simulated data programs were complete, the frequency response methods were developed. The frequency response method uses the cross-spectral density technique to generate magnitude and phase information. The program also generates the power spectral density functions. Noise contamination studies were made. The programs were used with the test project, HAVE DELAY, which was conducted at the USAF Test Pilot School. The frequency response program worked well up to a frequency of about 15 radians per second. Above 15 radians per second the programs suffer from additional noise and aliasing effects. Recommendations were made to study means of reducing noise effects to include anti-aliasing filters and noise processing schemes for digital data.

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EVALUATION OF A FREQUENCY RESPONSE TECHNIQUE FOR AIRCRAFT SYSTEM IDENTIFICATION

THESIS

Presented to the Faculty of the School of Engineering

of the Air Force Institute of Technology

Air University

In Partial Fulfillment of the

Requirements for the Degree of

Master of Science

by

Allan T. Reed

Capt USAF

Graduate Aeronautical Engineering

October 1985

Approved for Public Release; Distribution Unlimited

Preface

This thesis describes a method used at the Air Force Test Pilot School of identifying aircraft parameters via frequency response analysis techniques. This method uses various input and response signals from the aircraft and transforms these signals to generate frequency response data (Bode Plots). Lower order equivalent systems were fit to the frequency response data and equivalent aircraft parameters were determined. The method was developed using simulated aircraft time histories which were analytically derived. Flight testing was later accomplished which verified and supplemented the analysis.

The work on this thesis was accomplished through the Joint Air Force Institute of Technology/Test Pilot School (AFIT/TPS) Program.

I wish to express my gratitude to my thesis advisors Major (Dr.) James T. Silverthorn and Dr. Robert A. Calico. Much credit also is due the USAF Test Pilot School for sponsoring a test program through which my analytic work could be tested and verified. I also wish to thank my wife, Deborah, for her support throughout the two year joint AFIT/TPS program.

Allan T. Reed

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List of Symbols

Abbreviation or Symbol	Definition	Units
AOA	Angle of Attack	radians
B	Control Matrix	---
BW	Bandwidth	rad/sec
CAS	Control Augmentation System	---
DAS	Data Acquisition System	---
deg	degrees	---
dB	decibels	---
FFT	Fast Fourier Transform	---
Fn	Nyquist Frequency (Max Indentifiable)	rad/sec
ft	feet	---
KIAS	Knots Indicated Airspeed	---
j	$(j^2 = -1)$	---
lb	pound	---
LOES	Lower Order Equivalent System	---
ms	millisecond	10^{-3}sec
MSL	mean sea level	---
PR	Pilot Rating	(Cooper-Harper)
PSD	Power Spectral Density	---
q	pitch rate	deg/sec
rad	radians	---
sec	second	---
T	discrete time interval	sec

Abbreviation or Symbol	Definition	Units
TM	Telemetry	---
TPS	Test Pilot School	---
YAPS	Yaw-Angle of Attack-Pitot Static	---
Gx	Spectral Density of Input	---
Gy	Spectral Density of Output	---
Gxy	Cross Spectral Density	---
(ω)	frequency	rad/sec
ω_{sp}	short period natural frequency	rad/sec
ω_{dr}	dutch roll natural frequency	rad/sec
x(t)	input time history	---
$X(\omega)$	Fourier Transform of Input Signal	---
y(t)	response or output time history	---
$Y(\omega)$	Fourier Transform of Output Signal	---
ΔT	Sampling Interval	sec
ζ	damping ratio	---
δ_e	elevator deflection	---

Abstract

This paper presents the results of a project which used frequency analysis methods applied to flight test data in order to identify aircraft parameters. Computer programs were developed to generate simulated flight test data so the frequency response programs could be tested using a noise free data source. Once the simulated data programs were complete, the frequency response methods were developed. The frequency response method uses the cross-spectral density technique to generate magnitude and phase information. The program also generates the power spectral density functions. Noise contamination studies were made and the frequency response program was modified to use a window-averaging technique to reduce the effects of noise as well as to generate a coherence function to display where poor correlation between input and output was occurring due to noise. The programs were used with the test project, HAVE DELAY, which was conducted at the USAF Test Pilot School. Lower order equivalent systems techniques were used to fit results from flight test to lower order systems. The frequency response program worked well up to a frequency of about 15 radians per second. Above 15 radians per second the program suffers from additional noise and aliasing effects. Recommendations were made to study means of reducing noise effects to include anti-aliasing filters and noise processing schemes for digital data.

EVALUATION OF A FREQUENCY RESPONSE TECHNIQUE FOR AIRCRAFT SYSTEM IDENTIFICATION

I. Introduction

This thesis describes a method of identifying aircraft dynamic parameters (natural frequencies, damping ratios, etc.) using a frequency response technique. These techniques were developed for use by the USAF Test Pilot School.

Background

As modern aircraft and the flight control systems become more advanced, the response of these aircraft become increasingly difficult to evaluate. Most modern aircraft no longer exhibit the classic response characteristics for which exists a large body of knowledge (i.e. short period, phugoid, dutch roll, roll, and spiral). This has made the determination of MIL-SPEC (Ref 3), compliance extremely difficult, if not impossible. Also many times the design engineer is more interested in how the aircraft responds during pilot-in-the-loop or operational type maneuvering than how that aircraft responds open loop.

When flight control systems played a minor role in determining overall aircraft response then open loop response was adequate. But with complex flight control systems which significantly alter the nature of the response, the flight test engineer needs to examine the total system response--that is, from the force the pilot applies to the control stick to the final aircraft response. Often only part of the system needs to be analyzed.

A frequency response analyzer is ideally suited for this problem. Unlike classical flight test techniques which require precise inputs and test parameters to be performed by the test pilot, frequency analysis methods only requires that the system be sufficiently excited for a sufficient amount of time (about 20-40 sec), over a sufficient bandwidth (about 10 rad/sec).

Frequency response analysis methods are also superior to classical techniques in that frequency response techniques can be used while the pilot is flying tasks which are not much different from tasks the pilot would be flying if he were employing the aircraft operationally. Typical tasks for frequency analysis techniques would be air refueling, air-to-air tracking, air-to-ground tracking, precision formation, or precision landing. Thus, the design engineer is analyzing the total aircraft system (airframe plus flight controls), and is doing so with the aircraft flying maneuvers similar to those flown operationally.

Current Flight Test Techniques

The United States Air Force Test Pilot School currently uses open loop testing to determine aircraft natural frequencies and damping ratios. The existing technique involve stabilizing the aircraft at the desired test conditions (trim shot), and then applying a test input, (release from sideslip, doublet, etc.), releasing controls, and then recording aircraft response. Obviously the response being evaluated is an open loop response and effects of the flight controls are not entirely present. Data reduction for this technique involves the examination of strip charts (if the aircraft is instrumented) to determine the natural frequency and damping ratios. Damping ratios are typically determined by a time ratio method or log-decrement technique. (Noninstrumented aircraft are evaluated by the pilot timing and counting the number of overshoots following the input to determine natural frequency and damping ratio).

There are several problems associated with the current techniques. Aircraft response is generally a combination of motions. In other words, motion about the pitch axis is virtually never just short period motion or just phugoid motion, but rather some combination of the two motions. Only by exciting an aircraft by introducing as initial conditions a motion which is some multiple of the desired eiqenvector can one get motion of a single mode as a response. Often a sinusoidal pulse or control doublet is performed by the pilot at a frequency which is very close to the natural frequency of the mode being analyzed. The current methods and frequency sweeps can provide an idea of the frequency response based upon flight test and data reduction. Classical techniques tell much about an aircraft. <u>But</u>, they are <u>free</u> response techniques--they tell what happens to the aircraft when disturbed from equilibrium, controls released, and resulting motion taking place. What really needs to be examined is the forced response--because the critical phases of flight such as air-to-air, air-to-ground, air refueling, formation, takeoff, and landing are forced responses (pilot-in-the-loop). Furthermore, modern aircraft are often so augmented with feedback for stability and control that the classic modes may not appear in the standard manner or may be surrounded by additional modes introduced by flight controls (feel and trim systems, etc.). As an example of inherent errors in the classic methods of exciting an aircraft in-flight, consider the case where the engineer wishes to record the free aircraft response to a doublet input and typical pilot realization of that elevator input as shown in Figure 1. Shown are the elevator input signals as typically flown. Obviously a discrepancy exists between the desired input and the resulting input, and therefore, uncertainties will exist as to the real significance of the recorded response to these signals.

The concept of examining the forced response of an aircraft is important as forced response is the basis for handling qualities evaluations. The forced response, also called mission oriented response or pilot-in-the-loop response (Ref 14), is what ultimately must be optimized in the design of flight control systems. Handling qualities discovered during evaluation of this forced response are a function of the total system, from the pilot applying stick inputs down to the airframe aerodynamics.

The aeronautical/control engineer seeks better methods to analyze and optimize the forced response of an aircraft. This thesis presents a method of analyzing aircraft via frequency response methods.

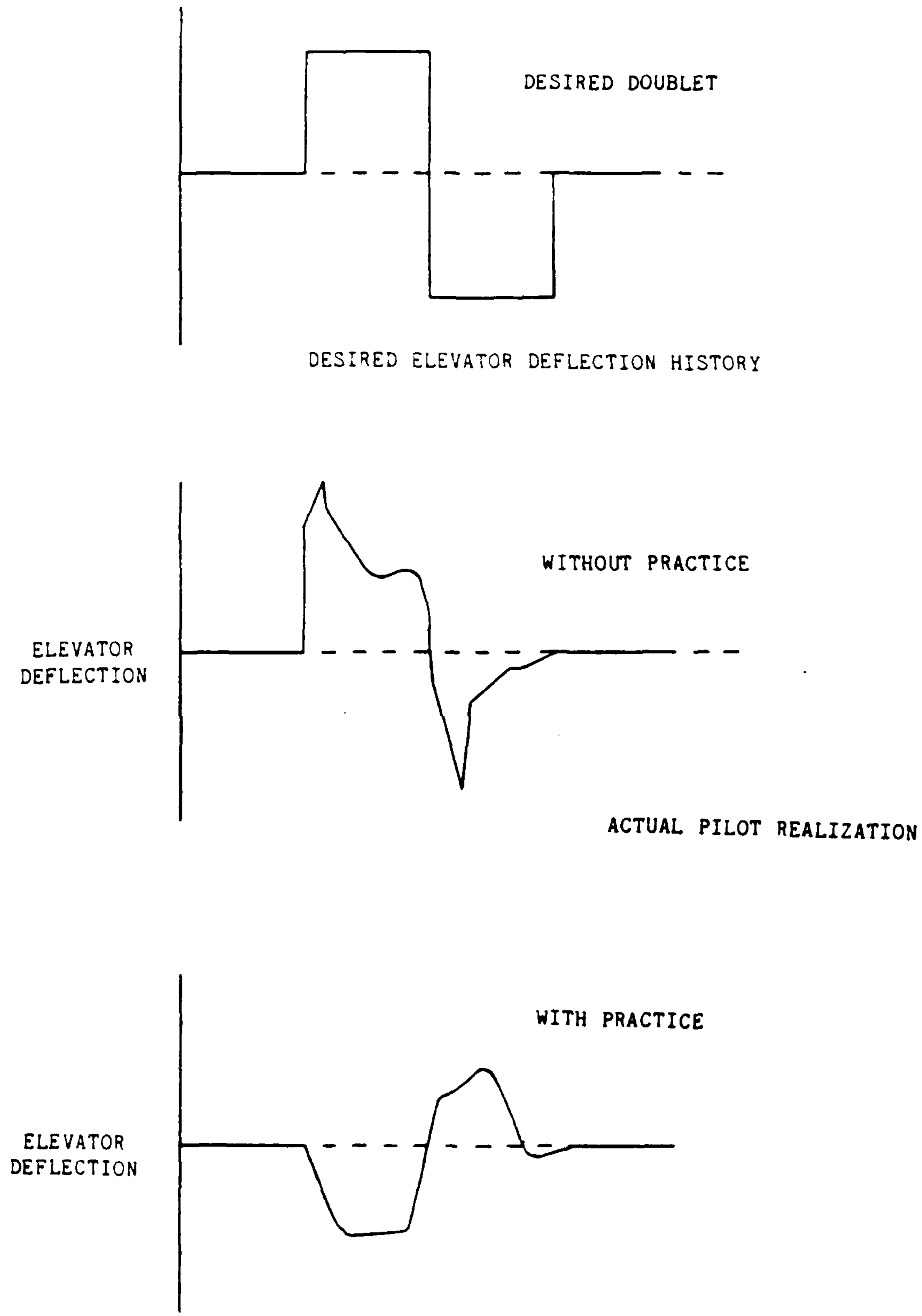


Figure 1 Desired and Typical Elevator Inputs

II. Frequency Response Method

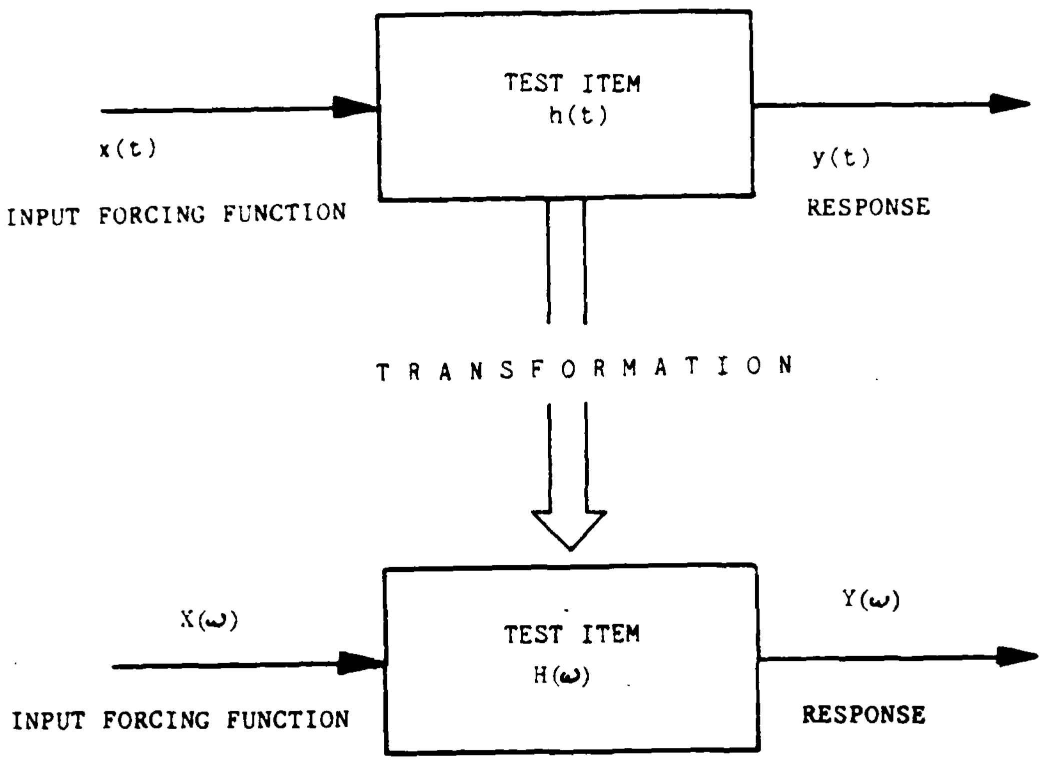
Frequency response methods require obtaining time histories of various inputs and the respective aircraft response. If these time histories can be appropriately transformed into the frequency domain then classic frequency domain analysis methods (i.e., Bode, amplitude and phase plots) serve the engineer in the analysis and optimization process. This concept is shown in Figure 2.

The choice of transformations is logically the Fourier Transform as shown in Figure 3, and since this project used flight test data which was digitally sampled, the transform which was used was a Discrete Fourier Transform or DFT.

Input Signal

The choice of input signals is perhaps unclear. If one demands a rigorous, precisely flown input signal, $x(t)$, then the same problem as the open loop method is encountered. The problem with the open loop test was that the pilot can never input precisely the desired signal (the test pilot cannot input a perfect step, a perfect doublet, etc.) (Figure 1)

Examination of the concepts behind the properties of the impulse function reveal an interesting property: if the input and response time signals are measured simultaneously and then Fourier Transformed, the resulting division of these two signals, i.e., $Y(\omega)/X(\omega)$ represents the transfer function between input and output. Thus, after a processing of test data from a single sampling of input and output a complete frequency response plot can be constructed. The



t = time

ω = frequency

h(t) = impulse response function

H(ω) = frequency response function

Figure 2 Frequency Analysis Concept

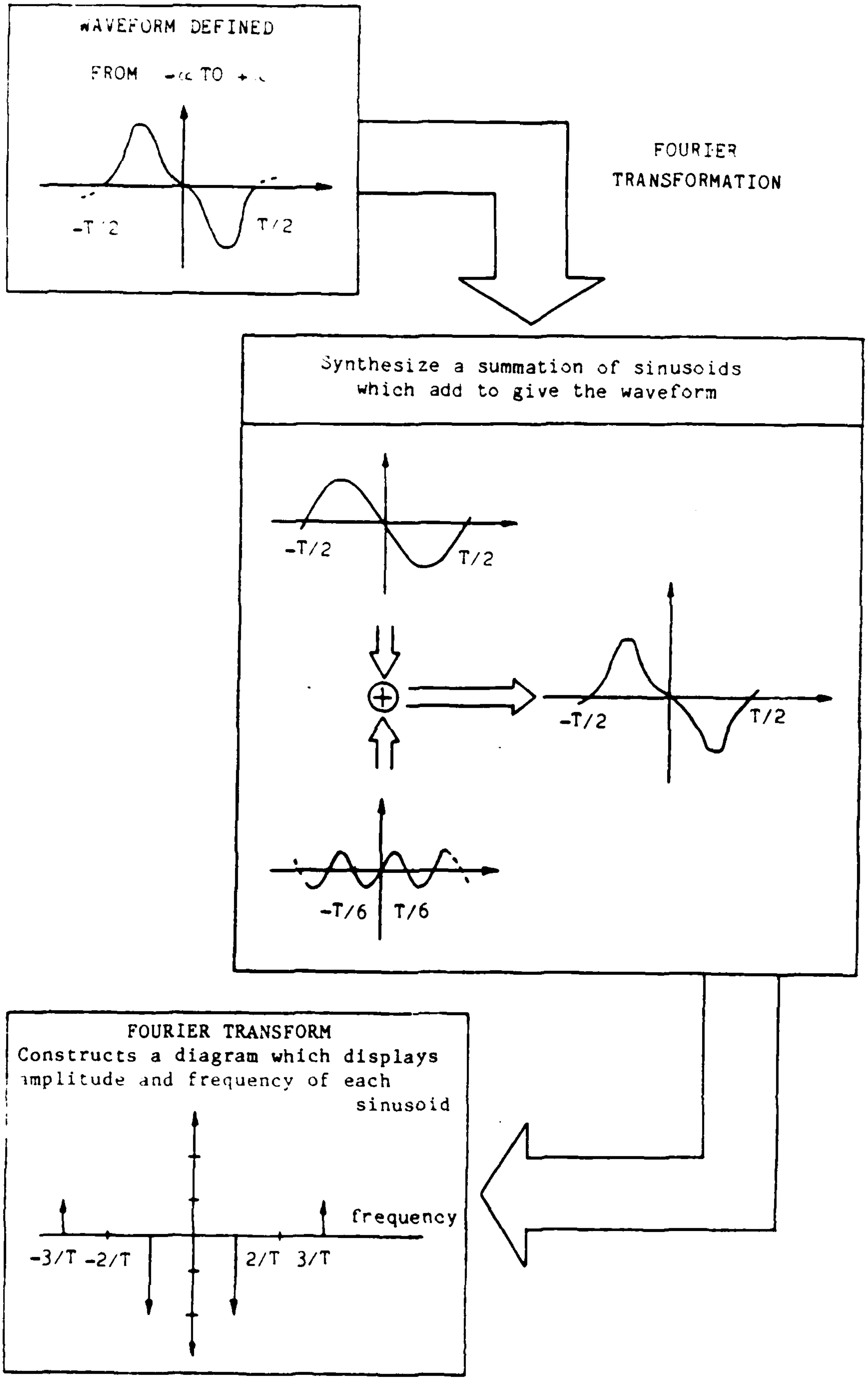


Figure 3 Interpretation of the Fourier Transform

only restriction to the input signal is that the frequency content must extend across the desired frequency range of interest. For aircraft flight dynamics, control engineers are most concerned with the frequency range 0.1 to 10 rad/sec.

The fact that the input signal must contain sufficient power in the frequency range of interest can be complied without any additional restrictions to the test pilot's input signal--thus, random input signals can be used if they contain an adequate amount of power in the range 0.1 to 10 rad/sec.

Thus the approach for this project was to digitally sample various inputs of interest and to simultaneously sample various outputs, use a digital Fourier Transform to convert signals from the time domain into the frequency domain and then to compute and plot the frequency response transfer function magnitude and phase angle for analysis.

Aliasing

Several problems arise in the frequency analysis of sampled data. One such problem is that of aliasing. Consider the two continuous waveforms shown in Figure 4: one of low frequency and one of high frequency. As long as the signal is sampled at least as fast as the theoretical limit (a minimum of two samples per cycle), the transform methods will correctly identify the signals. However, there are always practical limits as to how fast the signal can be sampled (i.e., 8 samples/sec, 16 samples/sec, etc.). A problem occurs if there are frequencies present in the signals which are higher than the maximum frequency which can be identified by the sampling rate.

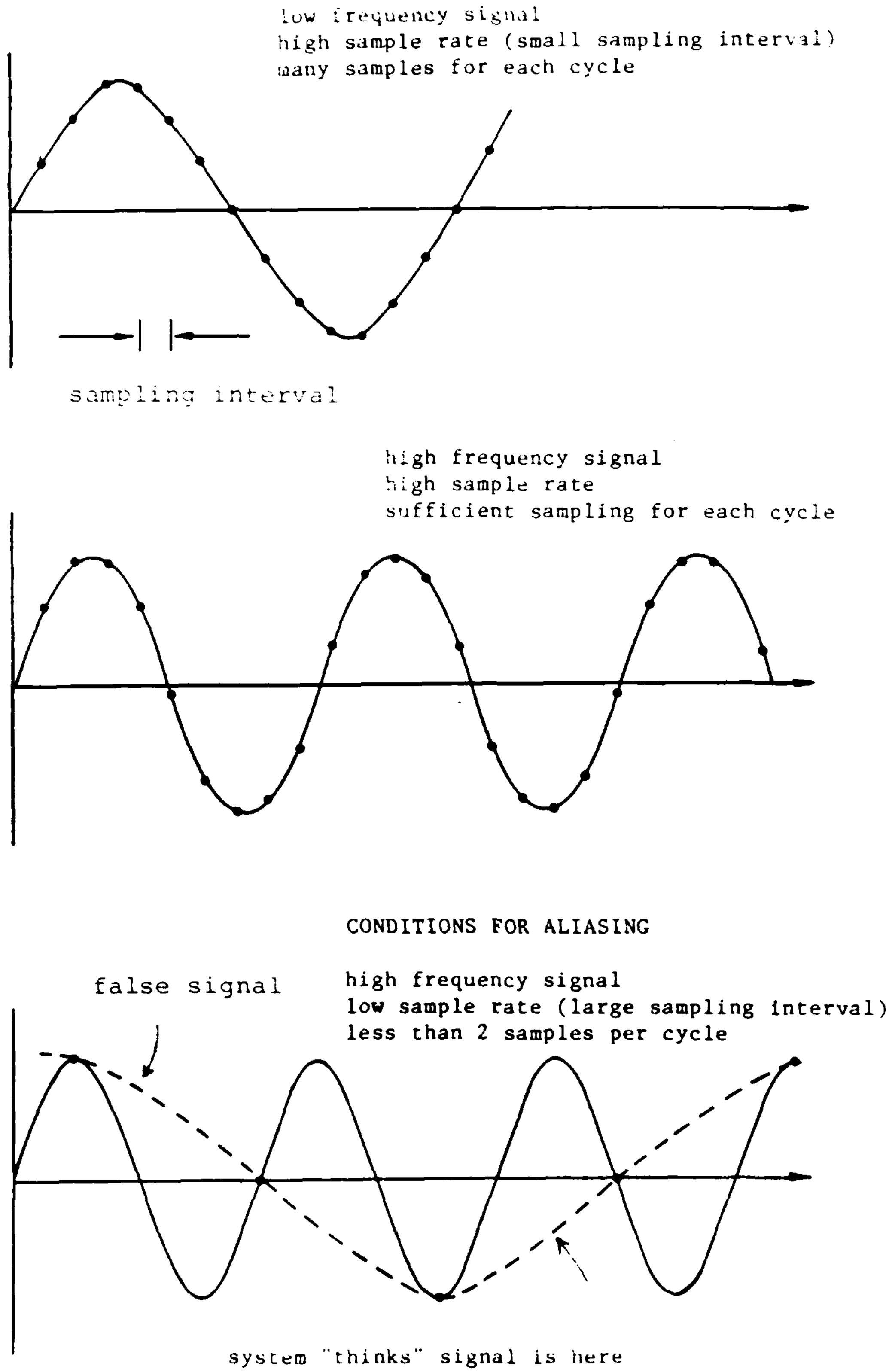


Figure 4 Concept of Aliasing

Figure 4 shows an example of the classic case of aliasing. The dotted line shown is an alias of the signal represented by the solid line.

This has a significant impact on the analysis of aircraft data. Given the frequency range of interest, one must ensure that the sampling rate is such that the highest frequency is sampled at least twice per cycle (the Nyquist rate). Aircraft flight dynamics generally deal with a frequency range of 0.1 to 10 rad/sec. Therefore a sampling rate is needed which gives at least two samples per cycle of a 10 rad/sec signal, i.e. sample at least twice during a cycle of 0.63 seconds time interval or once every third of a second. Thus, the bare minimum sample rate is three times a second ($Fn = 2\pi/2\Delta T$).

This is, however, a theoretical limit. By following proven engineering guidelines of five samples per cycle, one finds that one should sample the signals at approximately 0.13 sec ($Fn = 2\pi/5*2\Delta T$).

Sixteen samples per second should allow good analysis out to about 20 rad/sec. Some Test Pilot School aircraft have data acquisition systems (DAS) which sample 20 times per second which should allow analysis out to about 25 rad/sec. However, remember this discussion of aliasing assumes that one knows the highest frequency present in the data and can sample at an appropriate rate to prevent the higher frequencies from aliasing the lower frequencies. Since a basic linear system is assumed, the output spectrum will be limited to the same range as the input spectrum.

Once the proper choice of sampling rate is used, the aliasing will be reduced to effects which occur from noise in the data. If analysis of input and output spectrum show aliasing still to be a problem (i.e., due to nonlinear effects, buffetting, aerodynamic noise, etc.) then a low pass anti-aliasing filter and higher sample rate should reduce aliasing effects.

Window Effects

Another problem which arises in the method of frequency transformations of sampled data is the fact the input and output signals are not periodic (we are dealing with "random" inputs). The fact the signals are not periodic cause an effect known as "leakage".

If 20 sec of data are analyzed, the effect is as though one is looking at data through a 20 sec long rectangular window and neglecting everything which happened before the window started and everything which happened after the 20 sec window stopped.

In essence, multiplication of the data with the window in the time domain produce a convolution of the Fourier transform of the data window $W(\omega)$, with the Fourier transform of the data $X(\omega)$. Unfortunately, the rectangular data window has many undesirable features in the frequency domain. Namely, its flat top shape results in a multilobe shape in the frequency domain. This multilobe shape will allow energy to appear at these lobes (side lobe), thereby giving the impression that nearby frequencies are present when in fact they are not.

Consider the examples shown in Figures 5, 6, and 7. Figures 5 and 6 show periodic and non-peroidic signals. Figure 7 shows a signal which is periodic in the window 0-T sec. Upon transformation to the frequency domain, the positive frequencies would appear as shown.

As expected, the transformation of the 2sin3t waveform into the

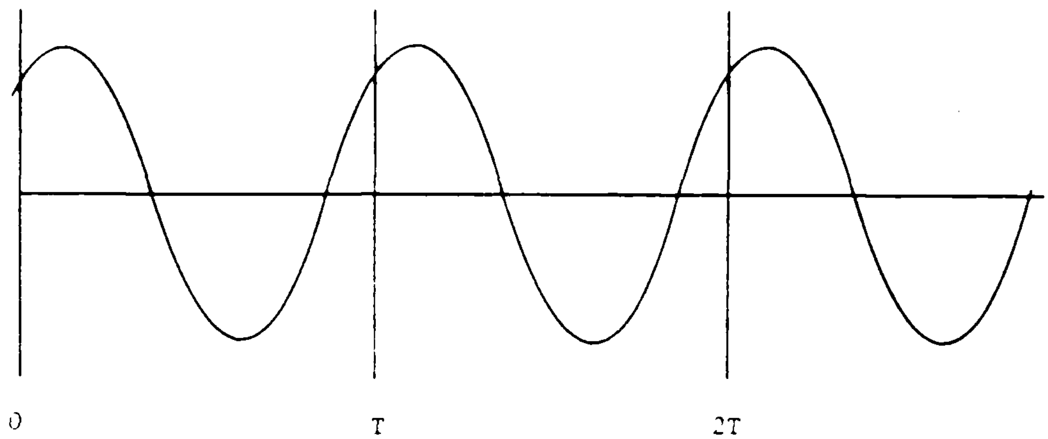


Figure 5 Periodic Signal in Window

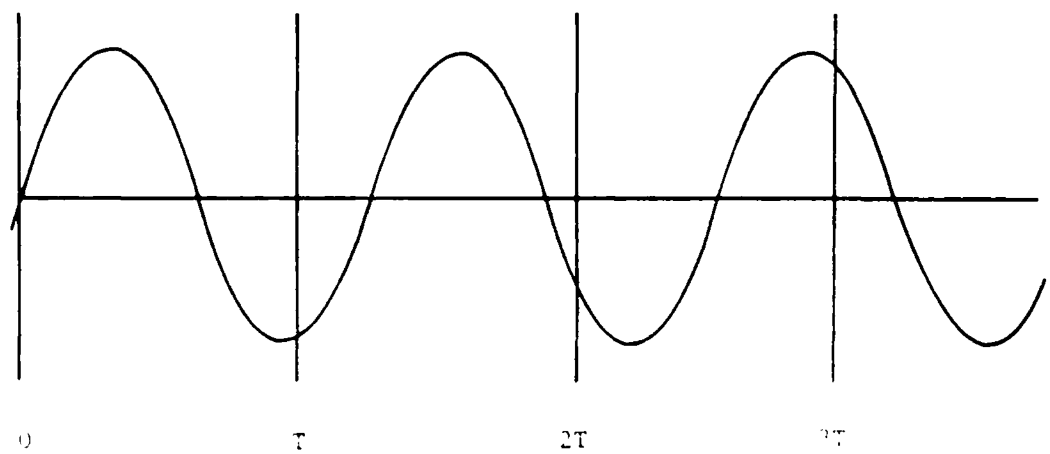


Figure 6 Non-Periodic Signal in Window

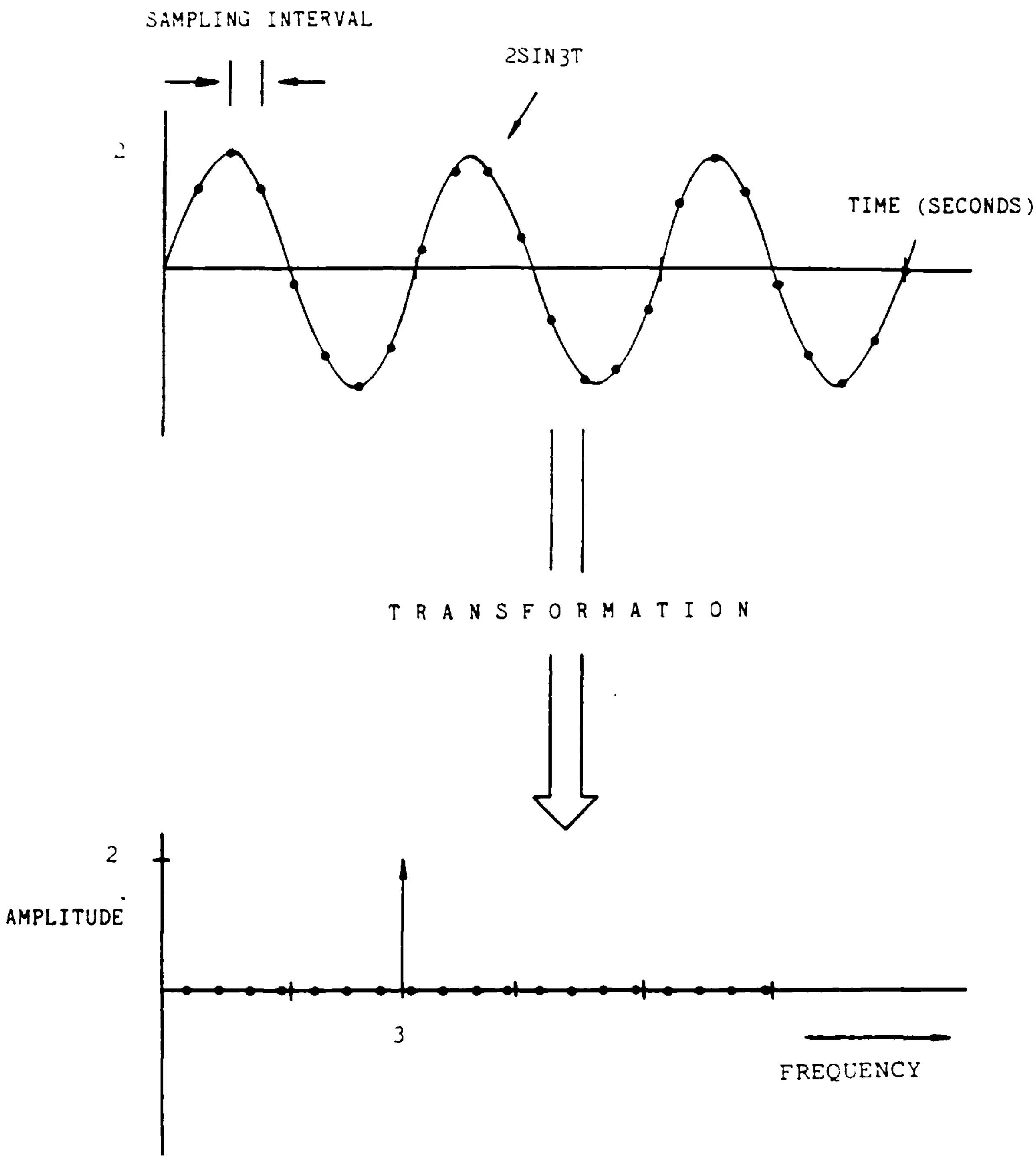


Figure 7 Transformation of Periodic Signal

frequency domain shows that the only signal present is at 3 rad/sec. Now consider the signal of Figure 8. This signal is not periodic in the T sec window. Notice that the 3 rad/sec signal does not appear as a single spike at 3 rad/sec but rather is smeared from about 2.5 to 3.5 rad/sec. Thus "leakage" has occurred through the side lobe of the data window w(t) because the data signal was not periodic in the window.

In aircraft frequency response analysis, virtually all of the signals (input and output) will be non-periodic, and therefore, one must account for leakage or else the frequency spectrum will be so smeared as to make frequency analysis impossible. The only other cure for the leakage problem is to take an extremely long sample size—with the limiting case being no leakage with an infinitely long window or sample size.

Window Selection

The practical way to cure the leakage problem is in the selection of a data window other than a rectangular window. The method of choosing the best window comes in analysis of each window's Fourier transform.

If a window is chosen which has a very narrow main lobe and virtually no side lobes, then the data can be passed through that window and the resultant frequency transform will not be smeared. One often used window is a Hanning window. The effect of windowing is shown in Figure 9.

Technically the data is now distorted and the effect would corrupt the transfer function. However, since both the input and

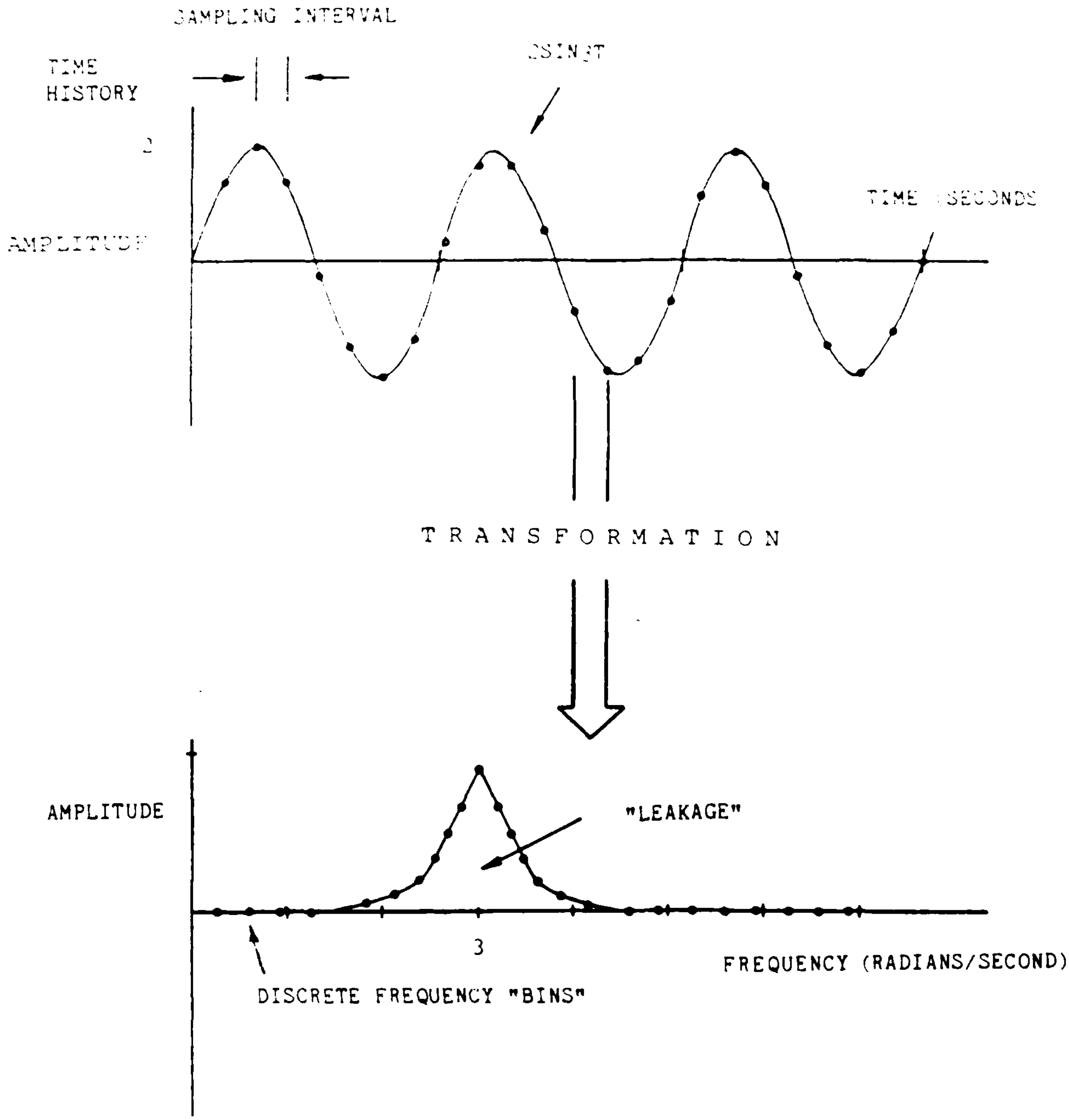


Figure 8 Transformation of Non-Periodic Signal

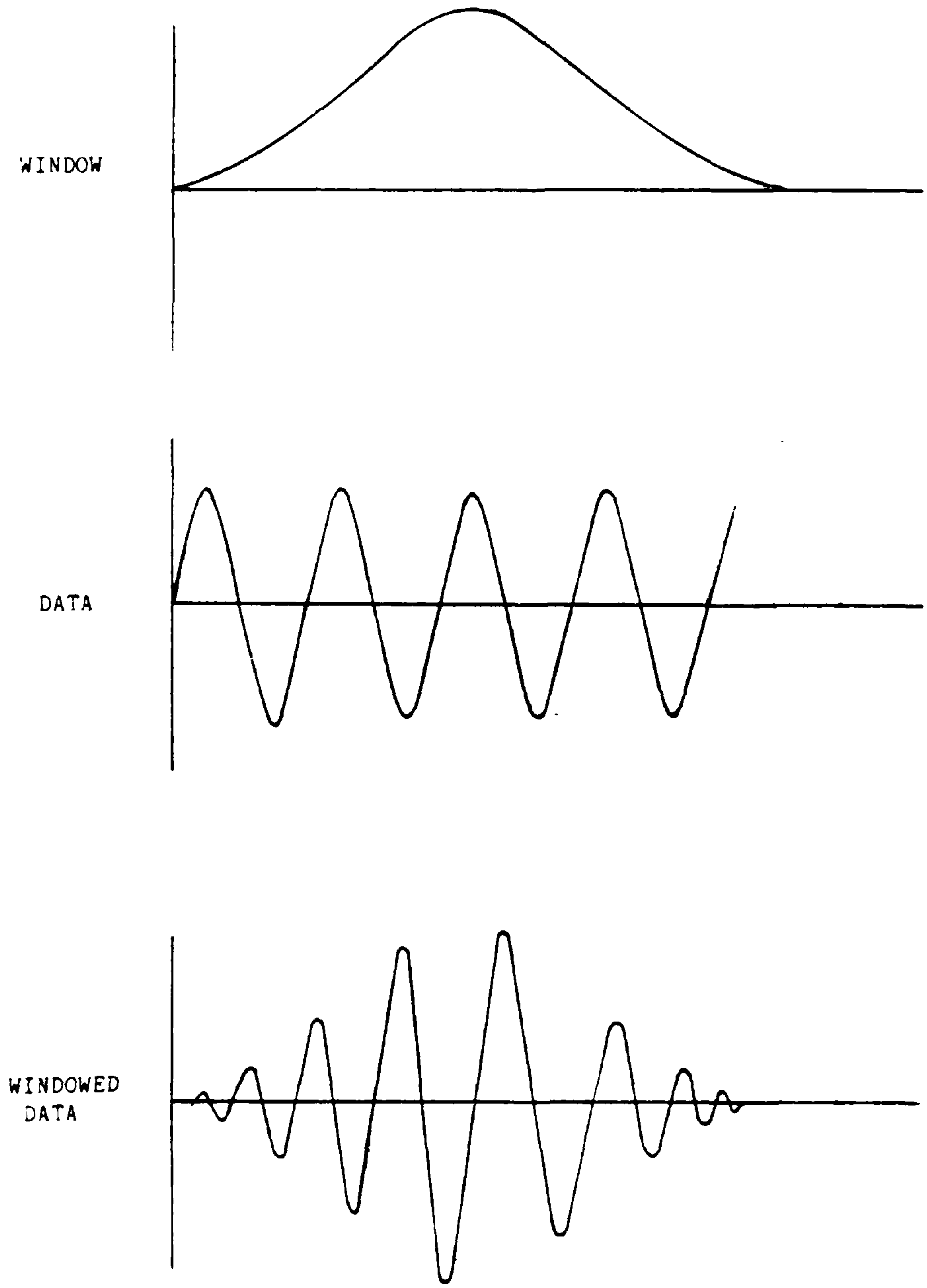


Figure 9 Effect of Windowing Data

output data signal must be "windowed", this distortion of windowing is removed in the calculation of the transfer function as long as the same window is applied to both input and output data.

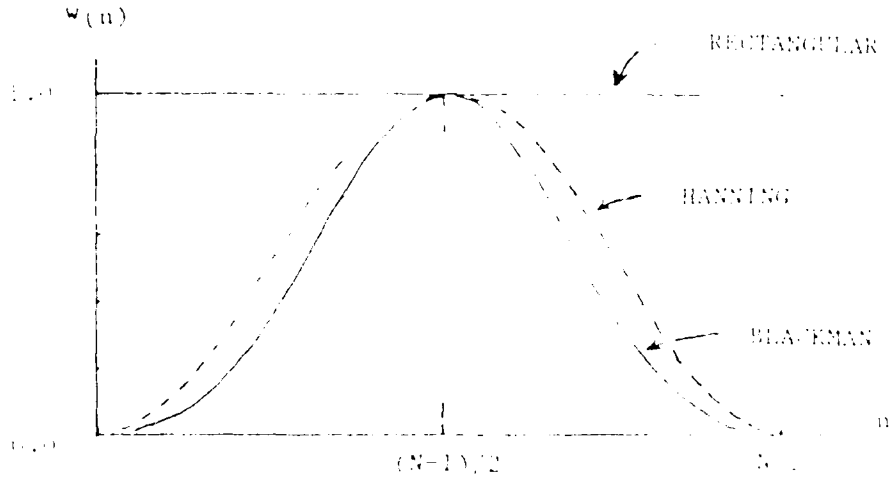
Figure 10 shows various windows and their respective Fourier transform. The question of which window is best for this application is difficult to answer since several factors are involved. The "specifications" of the Hanning window are that the first lobe is 23 dB down from the main lobe and each adjacent lobe falls off at -12 dB per octave (40 dB per decade).

Because of the presence of these sidelobes there is still some leakage as occurred with the rectangular window. The specifications of the original rectangular window are that the highest sidelobe is 13 dB down from the main lobe and each sidelobe falls off (decays) at 6 dB per octave. Thus the Hanning window is better than the rectangular window and one will acheive much more accurate frequency information if the input and output signals are passed through a window such as a Hanning window prior to the calculation of Fourier transform.

One desires a window with a very large dB drop between the main lobe and the first side lobe where the most leakage will take place. In addition each successive lobe should have a healthy rate of dB drop from the previous lobe. Otherwise there would be a minute leakage from signals far removed from that in questioned and this minute leakage would accumulate and distort the results.

The main lobe is also an important factor in selecting the optimum window. One desires a window with a very narrow main lobe. It should be no wider than the frequency resolution required for the aircraft

TIME DOMAIN REPRESENTATION OF WINDOWS



FOURIER TRANSFORM OF WINDOWS

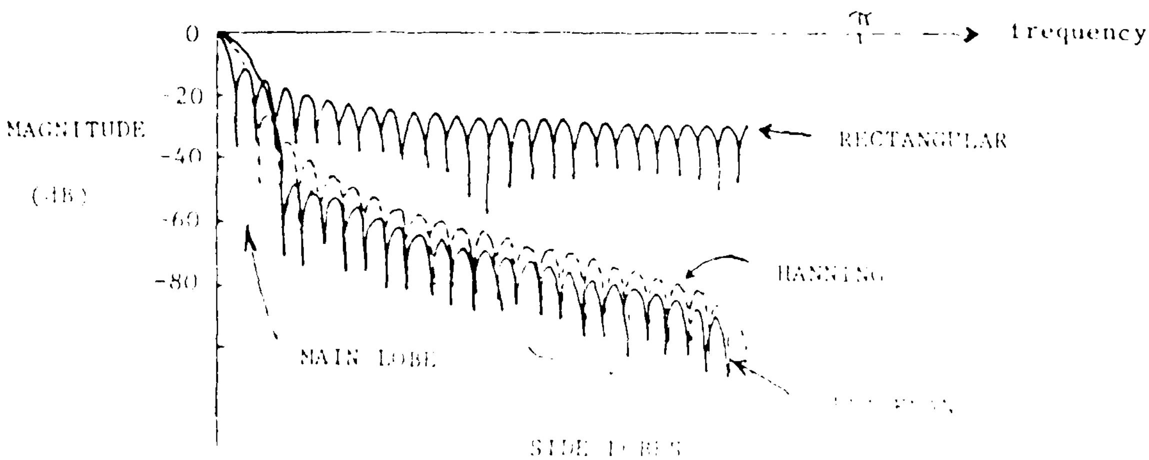


Figure 10 Fourier Transformation of Window

analysis. Table 1 contains a listing of various windows and specifications. The effects of windowing are always present, and one can almost always acheive better results by using a window other than the rectangular window. (i.e. no windowing)

Window Parameters

The characteristics of a particular weighting scheme, or window, depends in general on the number of data points used. As a comparison of windows, (Ref 7), Table 1 contains a list of some pertinent parameters based upon a sample size of 50 data points (N=50). The 3.0 and 6.0 dB bandwidth is indicated by frequency bin widths and indicate the points at which the gain of the shaped filter (window) is down 3 or 6 dB. The overlap correlation is based on a correlation computation given in Ref 7 and is a relative measure of the degree of correlation between random components in successive windows as a function of the amount of overlap.

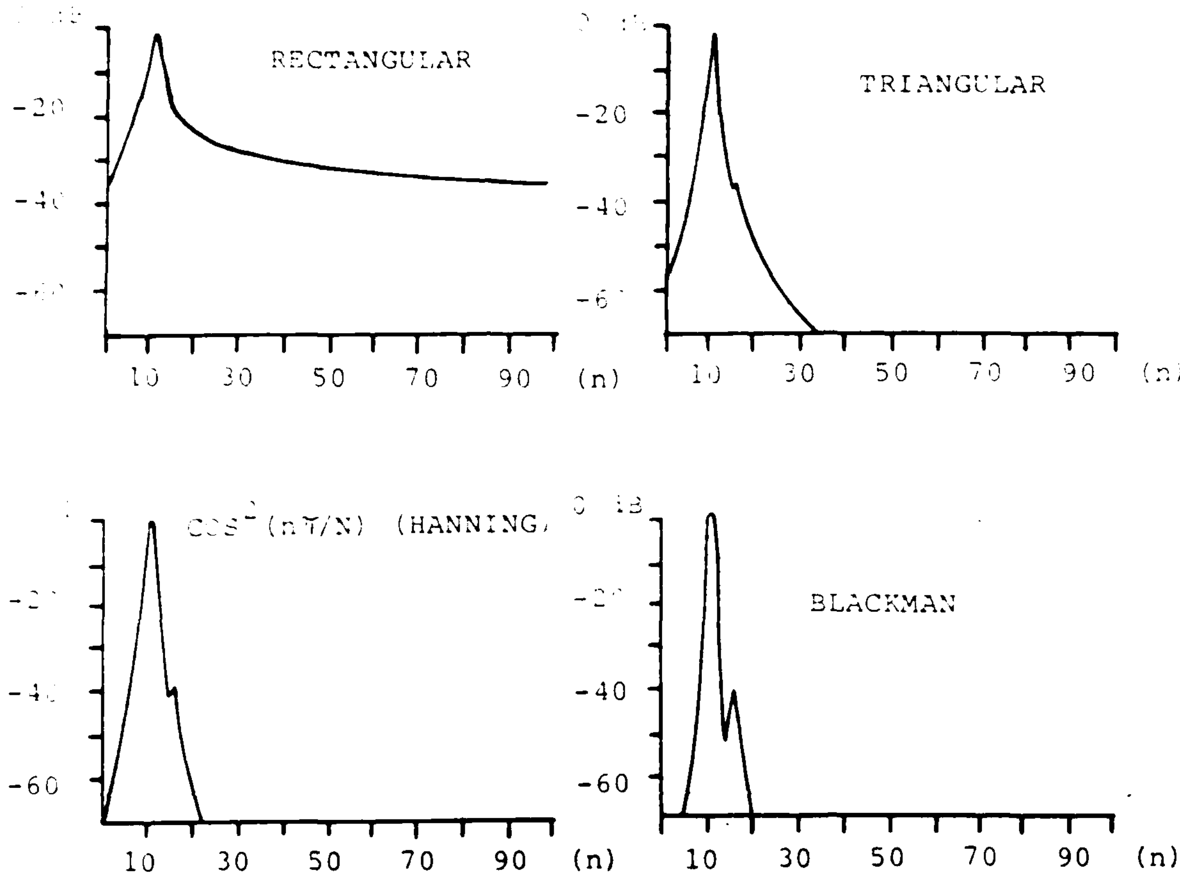
Another very good example of the effect of windowing data is to examine a signal which consists of only two frequencies. One frequency is dominant, the other is 40 dB lower in signal strength but is located very close tc the dominant signal. The effects of various windows are shown in Figure 11.

Thus, the type of window of filter shape chosen for the frequency transform of the time data has a significant impact on the detection of a weak signal located close to a strong signal. Good detection of these weaker signals comes from using windows or filters which have low sidelobe levels. However a characteristic of these filters is

TABLE 1

WINDOW SPECIFICATIONS

WINDOW (WEIGHTING)	HIGHEST SIDELOBE LEVEL (DB)	SIDELOBE FALL-OFF (DB/OCTAVE)	3DB BANDWIDTH	6DB BANDWIDTH	SCALLOPING LOSS	OVERLAP CORRELATION 75%	OVERLAP CORRELATION 50%
RECTANGLE	-13	-6	.89	1.21	3.92	75.0	50.0
TRIANGLE	-27	-12	1.28	1.78	1.82	71.9	25.0
HANNING (COS**2)	-32	-18	1.44	2.00	1.42	65.9	16.7
COS**3	-39	-24	1.66	2.32	1.08	56.7	8.5
HAMMING	-43	-6	1.30	1.81	1.78	70.7	23.5
BOHMAN	-46	-24	1.71	2.38	1.02	54.5	7.4
GAUSSIAN (ALPHA=3)	-55	-6	1.55	2.18	1.25	57.5	10.6
BLACKMAN	-58	-18	1.68	2.35	1.33	62.7	14.0
KAISER-BESSEL	-69	-6	1.74	2.44	1.02	53.9	7.4



SIGNAL	FFT bin	SIGNAL AMPLITUDE
1	10.5	1.00
2	16.0	0.01

Figure 11 Window Effect on Signal Detection

that as the sidelobe level goes down (less leakage) the mainlobe gets wider (poor frequency resolution).

In the work presented in here, the Blackman window was used to filter the input and output signals. The Blackman filter is easy to implement in the time domain (one line in a Fortran program) and has very good specifications in terms of sidelobe, sidelobe fall-off, scalloping loss, etc.

The Hanning window is typically used in linear system testing; namely vibrational analysis. The reason the Hanning window is popular is that its specifications are generally adequate for most applications and, it is very easy to implement in either the time domain or the frequency domain. Users of the Hanning window find it convenient to transform input and output signals into the frequency domain and then remove the leakage effects by smoothing the frequency data with the Hanning filter. Because of its simplicity and versatility the Hanning filter has been "hard-wired" into frequency analyzers-devices specifically design to perform very high speed spectrum analysis and identification. It should be noted however, that the selection of the "best" window or filter for most purposes assume a single pass of the entire input and output signals to calculate the various frequency functions Gx, Gy, Gxy, Hxy, etc. In reality, when the data is sectioned, processed, and averaged to obtain results then the apparent advantage of one window over another may be lost in the averaging (Ref 9).

III. FAST FOURIER TRANSFORMATION

Introduction

The mathematical foundation used in the frequency analysis of flight test data is the Fourier transform. As shall be shown, the Fourier transform is an integral part in the calculation of input and output density functions, (Gx, Gy), cross correlation function (Gxy), and the actual frequency response function (Hxy). The Fourier transform can be used for characterizing linear systems and for identifying individual frequency components of a continuous waveform. (see Figure 3).

A very good interpretation of the Fourier Transform is given by Brigham (Ref 5). The Fourier transform, in essence separates the waveform into a number of sinusoids at their respective frequencies. The sinusoids can be summed to return the original waveform. The Fourier transform is represented by the sinusoids. The time domain signal gives amplitude and time information about the signal whereas the frequency domain give amplitude and frequency information. The Fourier transform is given by:

$$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j2\pi\omega t}dt \qquad (1)$$

where $x(t)$ = waveform to be decomposed.

and $X(\omega)$ = Fourier transform of $x(t)$

In the case of flight test data, one does not have a mathematical expression representing the input signal. In other words, no expression exists which can be substituted into (1) to calculate the Fourier Transform of the input (or output) signals. Instead the input and output consist of a sequence of samples of measurment transducers, taken at a reqular interval, e.g. every .05 seconds. Thus instead of a continuous signal there is a series of sampled data points. If a total of N of these sample data points are obtained, each taken at T seconds apart (i.e. T = sample interval), then the Discrete Fourier transform is given as:

$$X(n) = 1/N \sum_{k=0}^{N-1} x(k)e^{-j2\pi nk/N} \qquad n=0,1,2,3,4,5.......N-1 \qquad (2)$$

The resulting Discrete Fourier transform, X(n), of the signal x(k) will contain both a real and an imaginary part.

An N-point series in the time domain will transform into an N/2 point series in the frequency domain as the last N/2 points will be nothing more than the mirror image (the minus frequencies values) of the first N/2 points. The Digital Fourier transform (DFT) as shown in (2) can be rewritten in the form:

$$X(n) = 1/N \sum_{k=0}^{N-1} x(k)W^{kn} \qquad \text{where } W = e^{-2\pi j/N} \qquad (3)$$

The direct calculation of (3) require the solution of a matrix of terms,

$$[X(n)] = [W^{kn}]\ [x(k)]$$

or if the matrix notation is expanded, using a sample size of four;

$$\begin{vmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{vmatrix} \qquad \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & W^1 & W^2 & W^3 \\ 1 & W^2 & W^0 & W^2 \\ 1 & W^3 & W^2 & W^1 \end{vmatrix} \qquad \begin{vmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \end{vmatrix}$$

Fourier Transform Points = matrix made of x time series
(i.e. frequency data) $e^{-j2\pi/N}$ terms data

The matrix of $e^{-j2\pi/N}$ terms makes use of the relationship $W^{nk} = W^{nk\mathrm{mod}(N)}$. nkmod(N) is the remainder after division of nk by N. Thus if N=4, n=2, and k=3 then $W^6 = W^2$. ($e^{-j3\pi} = e^{-j\pi}$) To perform the matrix multiplication shown would require 16 complex products and 16 complex additions. Or in general for N data points, N^2 complex multiplications and additions would be required. Cooley, Turkey, and others (Ref 5) have shown that it is possible to make efficient use of the symmetry present in the above matrix to reduce the number of calculations dramatically. These methods of computing the DFT are called Fast Fourier Transforms (FFT). Whereas an N point sequence would take N^2 calculations using conventional DFT's, the same sequence could be calculated using FFT techniques with only $(N/2)\log_2 N$ computations. Or;

N = 1024 DFT requires 1,048,576 complex multiplications

N= 1024 FFT requires 5120 complex multiplications

Figure 12 reflects these results. Thus the FFT greatly reduces the amount of processing time required to transform large amounts of data.

For $N=2^\gamma$, (γ = integer), the FFT algorithms are simply a procedure for factoring an N x N matrix into γ matrices each of which are also N x N. The γ matrices, will be highly symmetric and contain an extremely large number of zeros. FFT algorithms make use of this symmetry, the large number of zeros in the matrices, and the fact that certain patterns are even present for the data in "bit reversed" order to reduce the number of calculations involved. A number of FFT algorithms are available for performing a rapid DFT. A modification of an algorithm suggested by Brigham (Ref 5) is employed

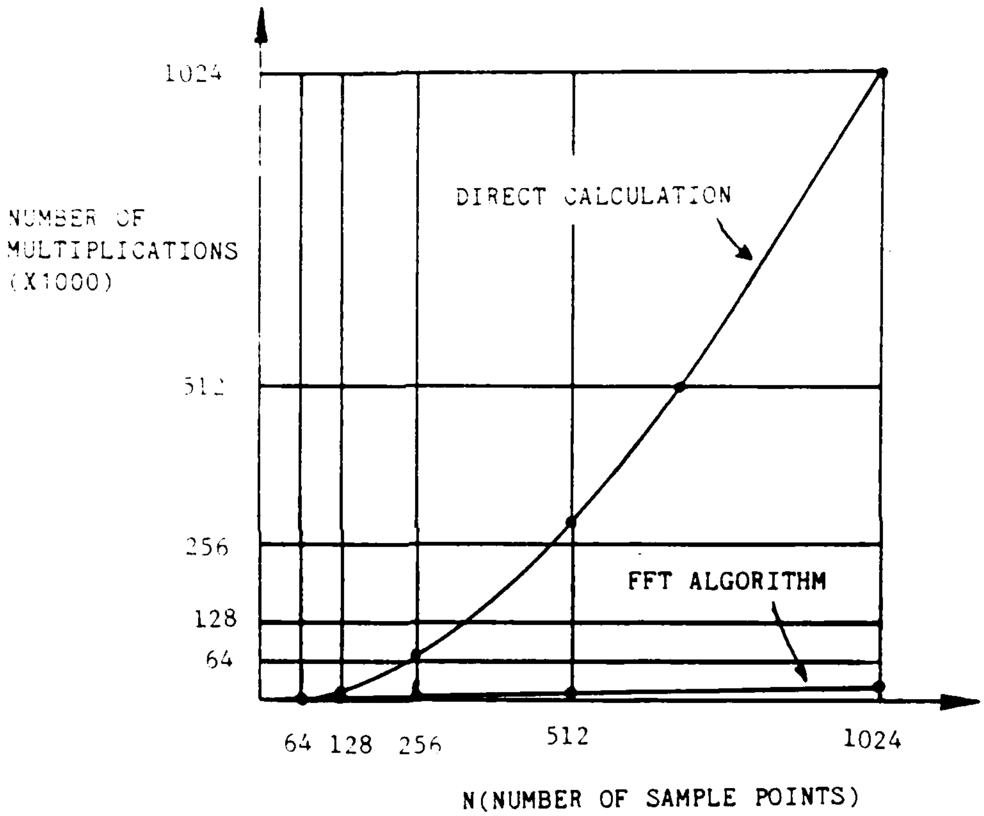


Figure 12 Comparison of Multiplications Required by Direct Calculation and FFT Algorithm

in the FFT Fortran subroutine shown in Figure 13.

Processing Real Data

To employ the subroutine of Figure 13, it should be recalled that the Fourier transform requires a complex input signal and therefore both a real and imaginary portion of the waveform must be provided. Real inputs signal should have their corresponding imaginary parts set equal to zero. However this approach is wasteful of both computer time and space. In the actual calculation of the transforms of input and output time series, one can compute the FFT of both signals simultaneously by inserting one signal into the imaginary component of the other, transforming, and then unscrambling the results. For example, if x(k) is the input signal and y(k) is the output signal, where the input and output signals are real signals, then one normally would set the imaginary part equal to zero and then use the FFT subroutine. But a new function, z(k), can be created such that

$$z(k) = x(k) + j\, y(k)$$

Through the use of the linearity property of the Fourier transform and by decomposing frequency signals into their real and imaginary parts, it can be shown (Ref 5) that the input and output signal can be recovered in the transformed state by the following;

$$X(n) = (R(n)/2 + R(N-n)/2) + j\,(IM(n)/2 - IM(N-n)/2)$$

$$Y(n) = (IM(n)/2 + IM(N-n)/2) - j\,(R(n)/2 - R(N-n)/2))$$

where $Z(n) = R(n) + jIM(n)$

Thus since both the input and output signals are real, they can be transformed at the same time with virtually a 50% savings in

```
C  ************************************************
C   SUBROUTINE FFT CALCULATES THE FOURIER TRANSFORM
C   OF THE INPUT AND RESPONSE SIGNALS
C
      SUBROUTINE FFT(XREAL,XIMAG,N,NU,J)
C
      DIMENSION XREAL(N), XIMAG(N)
C  ARRAY XREAL CONTAINS REAL PART OF TRANSFORM
C  ARRAY XIMAG CONTAINS IMAGINARY PART OF TRANSFORM
C   INPUT DATA IS DISTROYED
C   N IS NUMBER OF DATA POINTS (INTEGER POWER OF 2)
C   NU IS POWER OF 2 SUCH THAT N=2**NU
C   J = +1 GIVES FORWARD TRANSFORM (-1 GIVES INVERSE)
      N2=N/2
      NU1=NU-1
      K=0
      DO 100 L=1,NU
102   DO 101 I=1,N2
      P=IBITR(K/2**NU1,NU)
      ARG=6.283185*P/FLOAT(N)
      C=COS(ARG)
      S=SIN(ARG)
      K1=K+1
      K1N2=K1+N2
      TREAL=XREAL(K1N2)*C+XIMAG(K1N2)*S
      TIMAG=XIMAG(K1N2)*C-XREAL(K1N2)*S
      XREAL(K1N2)=XREAL(K1)-TREAL
      XIMAG(K1N2)=XIMAG(K1)-TIMAG
      XREAL(K1)=XREAL(K1)+TREAL
      XIMAG(K1)=XIMAG(K1)+TIMAG
      K=K+1
101   CONTINUE
      K=K+N2
      IF(K .LT. N) GO TO 102
      K=0
      NU1=NU1-1
      N2=N2/2
100   CONTINUE
      DO 103 K=1,N
      I=IBITR(K-1,NU)+1
      IF (I .LE. K) GO TO 103
      TREAL=XREAL(K)
      TIMAG=XIMAG(K)
      XREAL(K)=XREAL(I)
      XIMAG(K)=XIMAG(I)
      XREAL(I)=TREAL
      XIMAG(I)=TIMAG
103   CONTINUE
      IF (J .EQ. -1) J=N
      DO 104 I=1,N
      XREAL(I)=XREAL(I)/J
      XIMAG(I)=XIMAG(I)/J
104   CONTINUE
      RETURN
      END
C
C  FUNCTION IBITR  PERFORMS BIT REVERSING FUNCTION
C
      FUNCTION IBITR(J,NU)
      J1=J
      IBITR=0
      DO 200 I=1,NU
      J2=J1/2
      IBITR=IBITR*2+(J1-2*J2)
      J1=J2
200   CONTINUE
      RETURN
      END
```

Figure 13 FFT Computer Program

computational time. The only penalty is the unscrambling routine which must be employed to separate the two transforms. This concept was used in the the software developed for this project.

IV. FREQUENCY ANALYSIS PARAMETERS

Introduction

To analyze a system in the frequency domain, several parameters are required. One needs to generate the Power Spectral Density (PSD) of the input and output signals. The PSD of the input, Gx, is important as it allows one to examine the frequency content (spectrum) of input signal to tell at which frequencies the input signal contains the bulk of its power. For proper frequency response analysis, it is important that the system in question be excited across a wide enough bandwidth. Likewise a plot of the spectrum of the output signal, Gy, or PSD of the output would show the magnitude of the frequencies found in the output.

Quantities to be calculated in the frequency response analysis are:

1. The Power Spectral Density (PSD) of the input signal, x(t). The PSD of the input, annotated Gx, is sometimes called auto spectral density function or normalized power spectrum.
2. The Power Spectral Density of the output signal, y(t), is annotated as Gy.
3. The Cross Spectral Density function (CSD) relates the input and output time signals together: i.e. the output function y(t) and the forcing function x(t). The CSD of these two signals is denoted as Gxy, and the value of Gxy is used in the calculation of the Frequency Response Function and the coherence function.
4. The Frequency Response Function, or often referred to as the transfer function, will be denoted by Hxy.

5. The coherence function, γ^2, will be calculated and is helpful in analyzing systems in the presence of noise.

Armed with the five pieces of information listed above, Gx, Gy, Gxy, Hxy, and γ^2, one can now provide a detailed analysis of a system (i.e. aircraft, flight controls, etc). Below is a description of each of the terms and how they are generated from flight test data for this project.

POWER SPECTRAL DENSITY FUNCTIONS (Gx and Gy)

The input power spectral density, Gx, is the magnitude squared of the complex coefficients of the Fourier transform of x(t). Similarly, Gy, is the magnitude squared of the complex coefficients of the Fourier Transform of y(t). For example, if the Fourier Transform of x(t), evaluated at a frequency =2 n/N is written as:

$$x(\omega_i) = a_i + j\ b_i$$

then $Gx(\omega_i) = X(\omega_i)X^*(\omega_i)$

where $X^*(\omega_i) = a_i - jb_i$

Therefore $\quad Gx(\omega_i) = (a_i)^2 + (b_i)^2 \qquad (4)$

Similiarly, if

$$Y(\omega_i) = c_i + jd_i$$

then $\quad Gy(\omega_i) = (c_i)^2 + (d_i)^2 \qquad (5)$

The power spectral density is an easy way to examine a waveform to determine what frequencies are present and in what magnitudes. Since the analysis of aircraft flight control systems are generally most concerned with the frequency range of 0.1 to 10 rad/sec, one needs to examine the PSD of the forcing function x(t). Satisfactory results may seemingly be obtained from an unsatisfactory flight control system but close examination may show the flight control system was never excited in the frequency range where the problem exists. If one expects a problem near say, 9 rad/sec due to insufficient damping, and yet the input shows very little frequency content near 9 rad/sec occurred, then the pilot and the engineer might otherwise deduce that control is adequate in that frequency region.

Cross Spectral Density Function

The Cross Spectral Density function, Gxy, requires the Fourier transform of the input x, and output, y. As before, if

$X(\omega_i) = a_i + jb_i$ and

$Y(\omega_i) = c_i + jd_i$

then Gxy is defined as the cross conjugate product or

$$Gxy(\omega_i) = (a_i - jb_i)(c_i + jd_i)$$

expanding: $Gxy(\omega_i) = (a_i c_i + b_i d_i) + j(a_i d_i - b_i c_i)$

In the software designed for this project, the real part of Gxy is designated P and the imaginary part is designated Q. Therefore;

$$P_i = a_i c_i + b_i d_i$$

$$Q_i = a_i d_i - b_i c_i$$

or $Gxy(\omega_i) = P_i + jQ_i$

Since Gxy is a complex number, it contains both magnitude and phase information, given by:

$$\text{magnitude of } Gxy(\omega_i) = |Gxy(\omega_i)| = (P_i^2 + Q_i^2)^{1/2}$$

$$\text{phase of } Gxy(\omega_i) = \angle Gxy(\omega_i) = \tan^{-1}(Q_i/P_i)$$

Thus one can calculate the magnitude and phase of Gxy at any frequency, ω_i. The importance of this magnitude and phase information is shown below.

Frequency Response Function

The Frequency Response Function, $Hxy(\omega_i)$, is depicted by Figure 14. The frequency response function relates the input to the output in the frequency domain, and is defined as $Hxy(\omega_i) = Y(\omega_i)/X(\omega_i)$. The frequency response function is simply the Fourier transform of the impulse response function. Note that by using the previous definitions of Y and X:

$$Hxy(\omega_i) = (c_i + jd_i) / (a_i + jb_i)$$

Since Hxy is a complex quantity, one can calculate its magnitude and phase. Multiplying the numerator and denominator by the complex conjugate of the denominator, one obtains:

$$Hxy = (c_i + jd_i)/(a_i + jb_i) * (a_i - jb_i)/(a_i - jb_i)$$

$$Hxy = [(a_i c_i + b_i d_i) + j(a_i d_i - b_i c_i)] / (a_i^2 + b_i^2)$$

Therefore the phase of Hxy is:

$$\angle Hxy = \tan^{-1} (a_i d_i - b_i c_i)/(a_i c_i + b_i d_i)$$

Note that this is exactly the same expression obtained for the phase

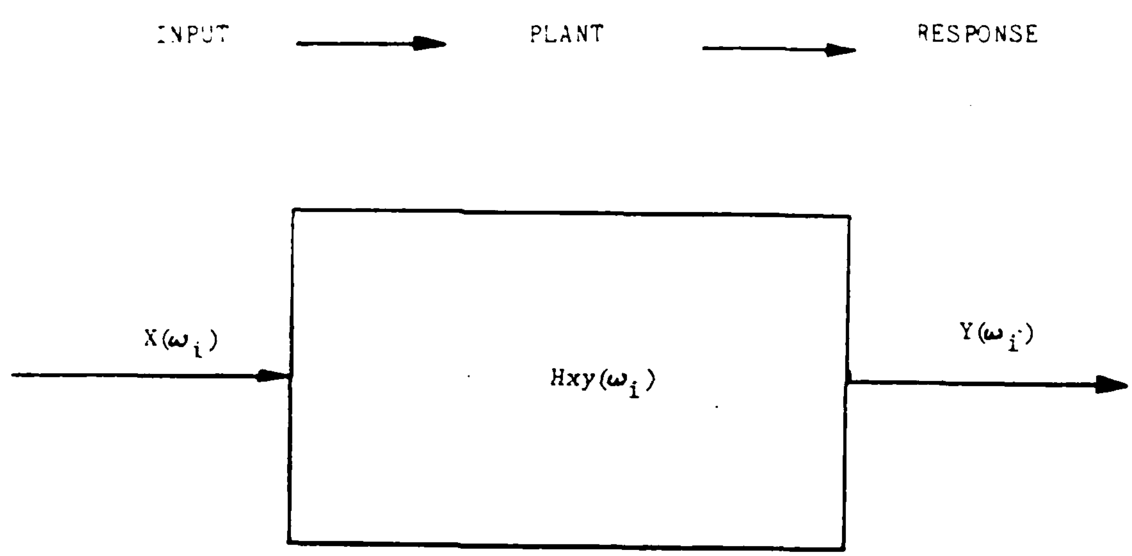


Figure 14 Frequency Response Function, Hxy

of Gxy. The magnitude of Hxy can be found as follows:

$$\text{Magnitude of Hxy} = (c_i^2 + d_i^2)^{1/2} \ / \ (a_i^2 + b_i^2)^{1/2}$$

$$= (Gy)^{1/2} \ / \ (Gx)^{1/2} \qquad (7)$$

This can be related to Gxy by expanding Gxy in terms of

a_i, b_i, c_i, and d_i.

$$\text{Magnitude of Gxy} = (P_i^2 + Q_i^2)^{1/2}$$

$$= [(a_i c_i + b_i d_i)^2 + (a_i d_i - b_i c_i)^2]^{1/2}$$

$$= [(a_i c_i)^2 + 2a_i b_i c_i d_i + (b_i d_i)^2 + (a_i d_i)^2$$

$$- 2a_i b_i c_i d_i + (b_i c_i)^2]^{1/2}$$

$$= [(a_i c_i)^2 + (b_i d_i)^2 + (a_i d_i)^2 + (b_i c_i)^2]^{1/2}$$

$$= [a_i^2(c_i + d_i)^2 + b_i^2(c_i + d_i)^2]^{1/2}$$

$$= [(a_i^2 + b_i^2)(c_i^2 + d_i^2)]^{1/2}$$

By equations (4) and (5), the product terms are Gx and Gy, and therefore

$$\text{Magnitude } Gxy(\omega_i) \quad = \quad [Gx(\omega_i)Gy(\omega_i)]^{1/2}$$

dividing both sides by Gx:

$$\text{Magnitude of } Gxy(\omega_i)/Gx(\omega_i) = Gy(\omega_i)^{1/2} \ / \ Gx(\omega_i)^{1/2} \qquad (8)$$

But since this is exactly the expression for the Frequency Response function, Hxy, as given in (7), $Hxy(\omega_i) = |Gxy(\omega_i)| / |Gx(\omega_i)|$

Thus there is no need to directly calculate the frequency response function by use of (7). Instead the magnitude of the frequency response function can be obtained from the ratio of the magnitude of Gxy and Gx. The phase angle of Hxy is simply the phase angle of Gxy. The processes for the calculation of PSD, CSD, and Frequency Response functions are shown in Figures 15, 16, and 17.

NOISE REDUCTION TECHNIQUES

The major source of error in the frequency response plots comes from high frequency signals corrupting the data because of the aliasing phenomenon and from the many sources of noise. Noise is present in the input and output signal measurement. Noise is also introduced in the digitizing process and is a function of the resolution of the measurement sensors.

For this program the coherence function was used to provide information to the user at which frequencies the response was least corrupted by noise. In addition, a noise reduction algorithm was added to the basic frequency response program to reduce the effects of noise. This technique and the coherence function are described below.

With the transfer function measured using the method previously discussed, the coherence function denoted (γ^2) can be computed. The coherence function, is defined as;

$$\gamma^2 = \frac{\text{(response power caused by applied input)}}{\text{(measured response power)}}$$

Ideally in the absence of noise the coherence function would be unity. When the measure response power is greater than the measured input power, e.g. because of some extraneous noise source is contributing to the output power, then the coherence value will be less than one.

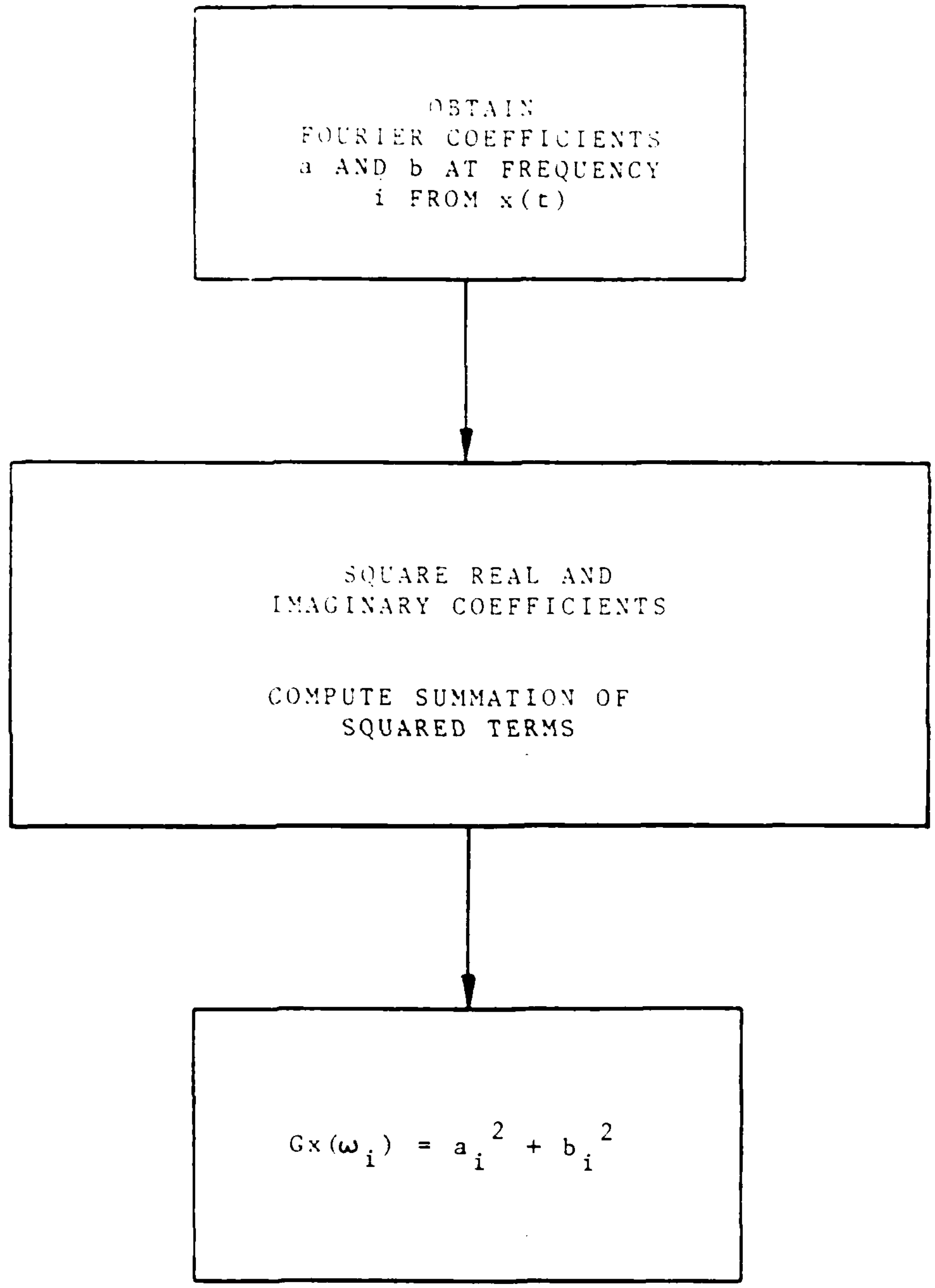


Figure 15 Power Spectral Density Process

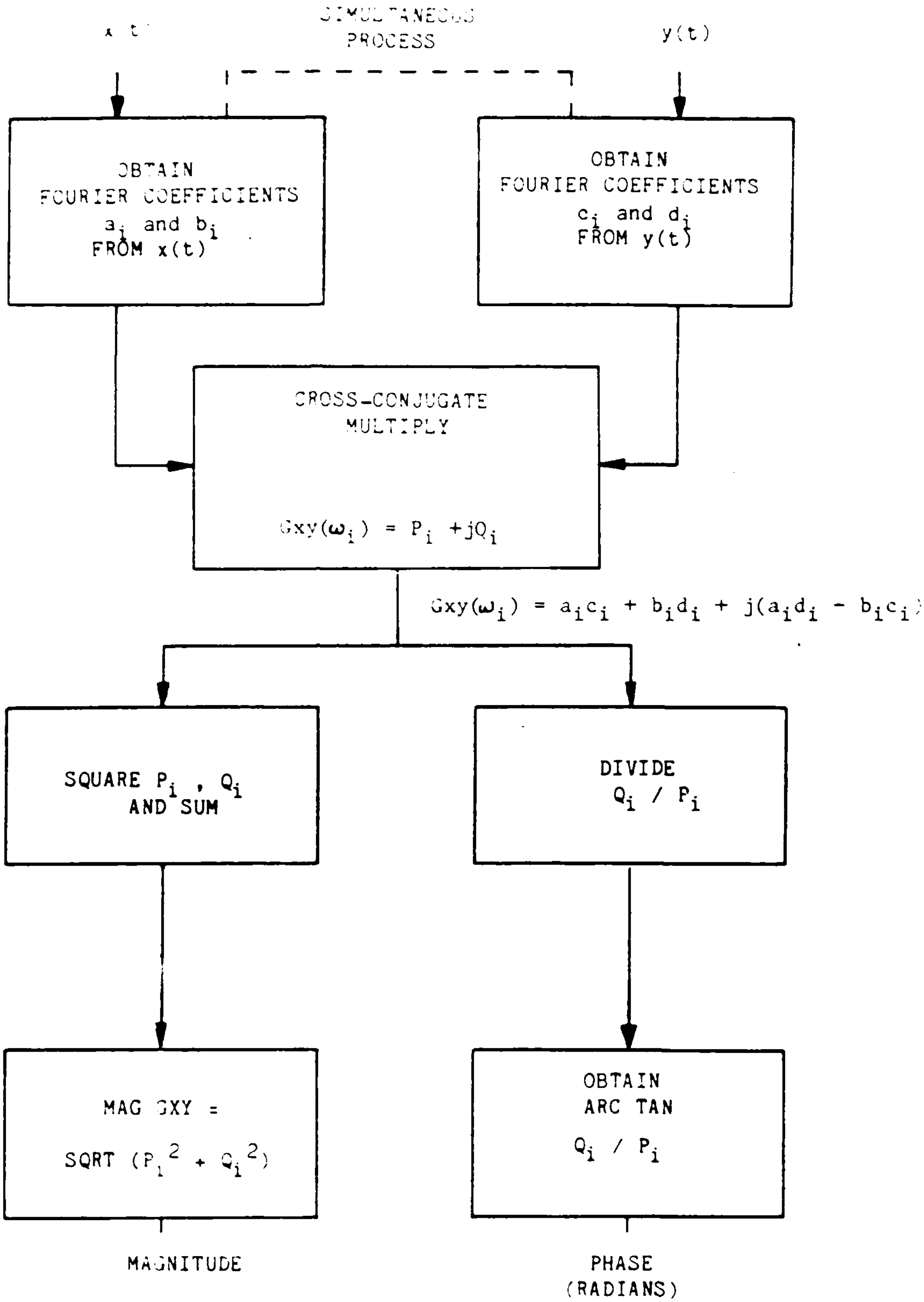


Figure [illegible] Cross Spectral Density Process

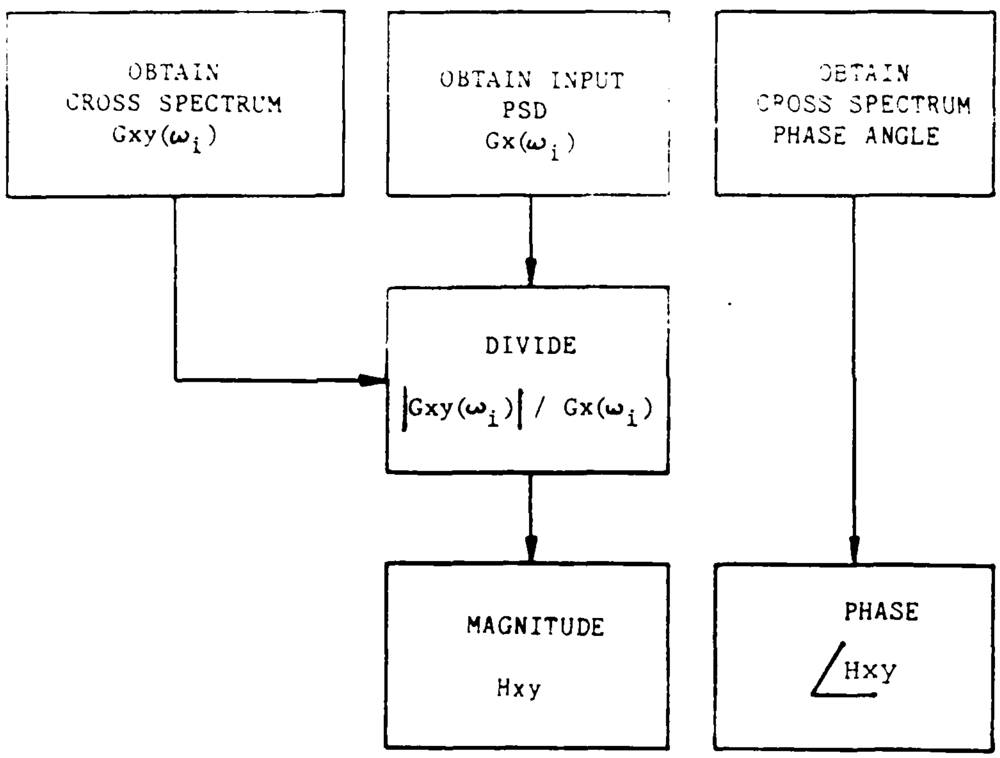


Figure 17 Frequency Response Process

This will be the case for those frequencies where the noise source adds power to the response signal.

Therefore the coherence function can be used to indicate the degree of noise contamination in the transfer function estimation. Because flight test data will inherently be corrupted by noisy measurements, the evaluation of γ^2 for the frequency range 0.1 to 10 rad/sec is extremely valuable. Figure 18 shows an example of the noise problem. Since one is most interested in identifying modal parameters from measured transfer functions, the variance of the estimates of those parameters is reduced in proportion to the amount of noise reduction in the measurements.

From Figure 18, one can compute $H(\omega)$ from the measured and transformed signals $X(\omega)$ and $M(\omega)$. $H(\omega)$ is defined as the ratio $Y(\omega)/X(\omega)$. Thus;

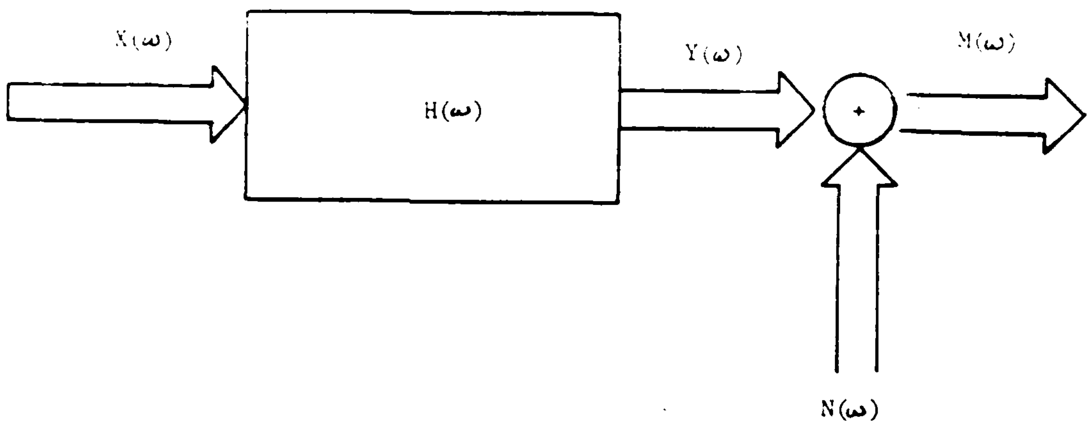
$M(\omega) = H(\omega)X(\omega) + N(\omega)$ or simply $M=HX + N$

The input power spectrum is Gxx and is defined as $Gxx = XX^*$ where * denotes the complex conjugate of the transform. The cross power spectrum is $Gmx = MX^* = (HX + N)X^* = HGxx + NX^*$

Consider averaging the quantity Gmx. The average value of Gmx from n different measurements is $\overline{Gmx} = 1/n \sum_{i=1}^{n} Gmx(i)$ where Gmx(i) is the ith measurement taken. Therefore $\overline{Gmx} = \overline{HGxx} + \overline{NX^*} = \overline{HGxx} + \overline{Gnx}$. Likewise Gxx and Gnx are defined in a similar manner. Therefore the actual frequency response of the system is given by;

$$Hmx = (\overline{Gmx}/\overline{Gxx}) - (\overline{Gnx}/\overline{Gxx})$$

notice that as the number of averages grows larger the noise term is reduced and the ratio Gmx/Gxx more accurately estimates the true



$X(\omega)$ = Fourier Transformation of Input Signal

$H(\omega)$ = Frequency Response Function

$Y(\omega)$ = Fourier Transform of Output Signal

$N(\omega)$ = Noise

$M(\omega)$ = Fourier Transform of Measured Signal

Figure 18 Measurement of Signal Plus Noise transfer function. A similar analysis is presented in Ref 6 and 12 for the frequency response of structures.

Figure 19 shows a frequency response of roll rate to stick force for a YA-7D (Ref 10). Figure 19 contains no averaging techniques and notice how at the higher frequencies much uncertainty exists as to the exact nature of the magnitude and phase information. Notice especially how the phase information is adversely affected by noise. Figure 20 is a frequency response plot of stick force to roll rate using the same data as was used to generate Figure 19. The program used to generate Figure 20 used the same basic techniques as the program used to generate Figure 19, however instead of examining a single 512 data point sequence, the program divided the 512 data points into seven 128 data point segments as shown in Figure 21.

While some frequency resolution was lost using this technique, the resolution was still high enough for aircraft parameter determination. More importantly, the averaging of the seven segments of data significantly reduced the scatter of the magnitude and phase curves especially in the lower range of frequency. This technique has now provided usable magnitude and phase information in the frequency range of interest, namely 0.1 to 10 radians per second. Larger sample sizes with even more segments would reduce noise even further. Notice that even with average techniques the curves are not usable beyond about 15 radians per second. This is due to the aliasing problems described earlier. It is also a function of the sampling rate. Engineering experience shows that reliable frequency information is not achieved at the Nyquist limit of 2 samples per cycle. Useful

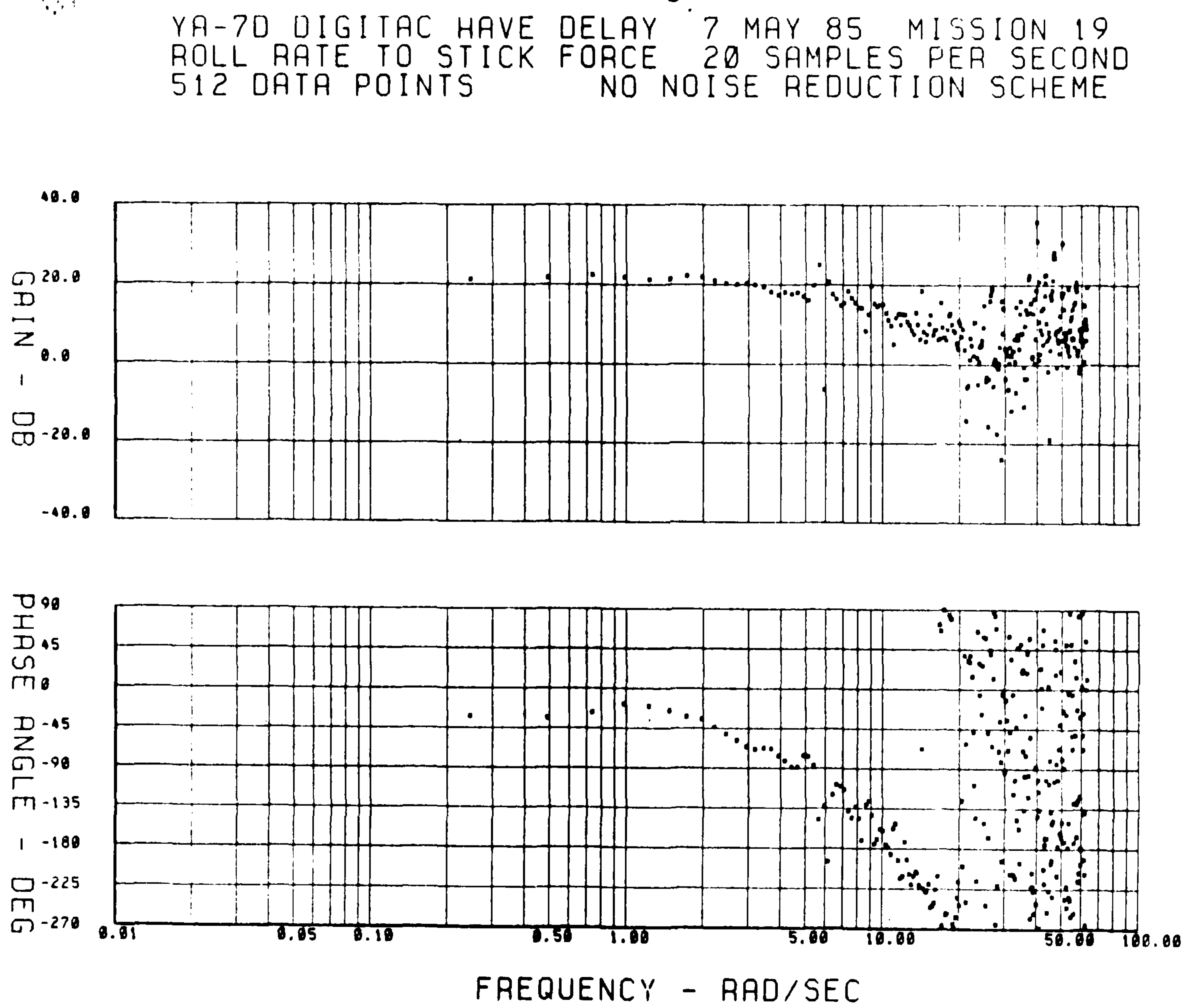


Figure 19 Frequency Response for configuration HD-000-055 (no averaging)

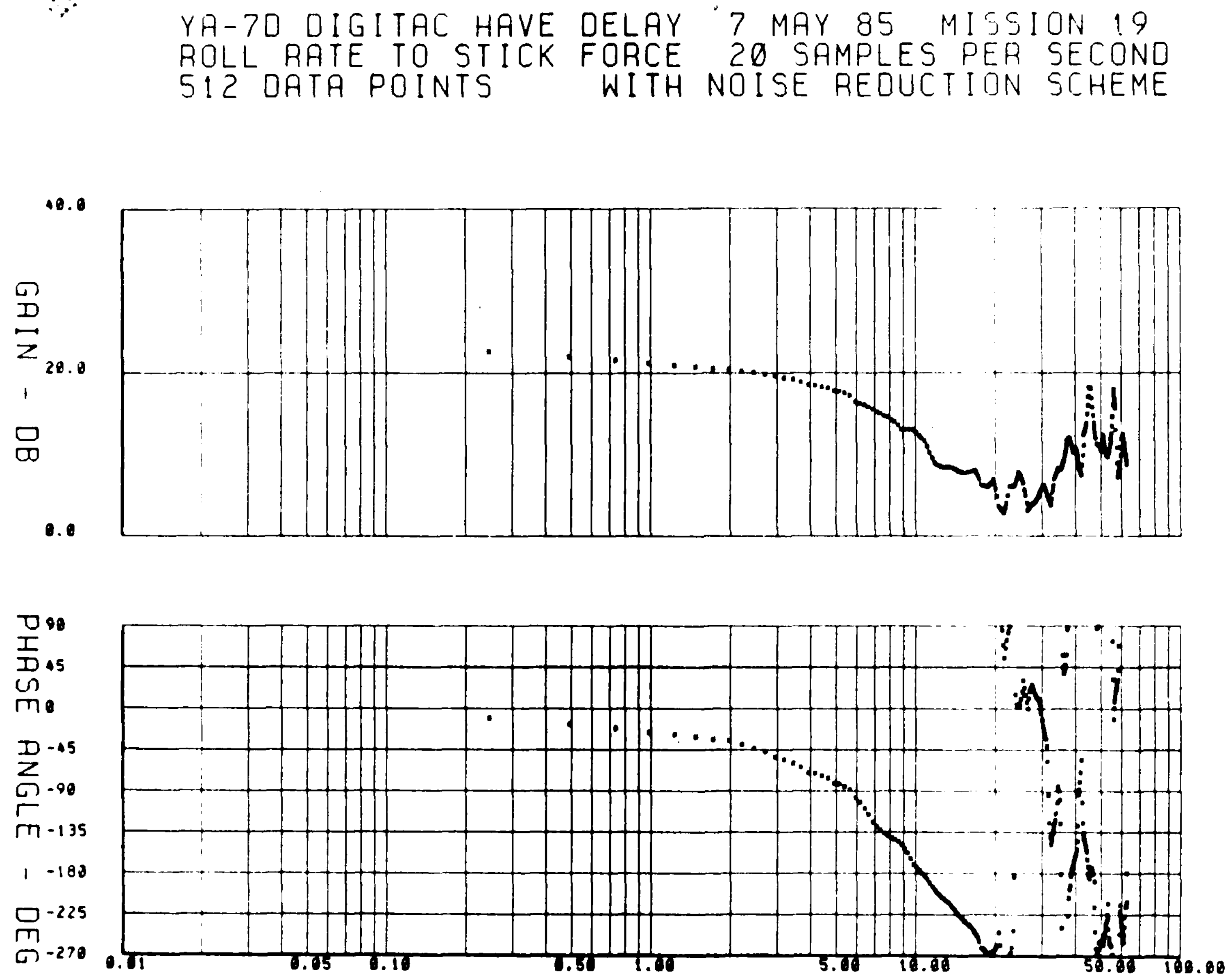


Figure 20 Frequency Response for configuration HD-000-055 (averaging)

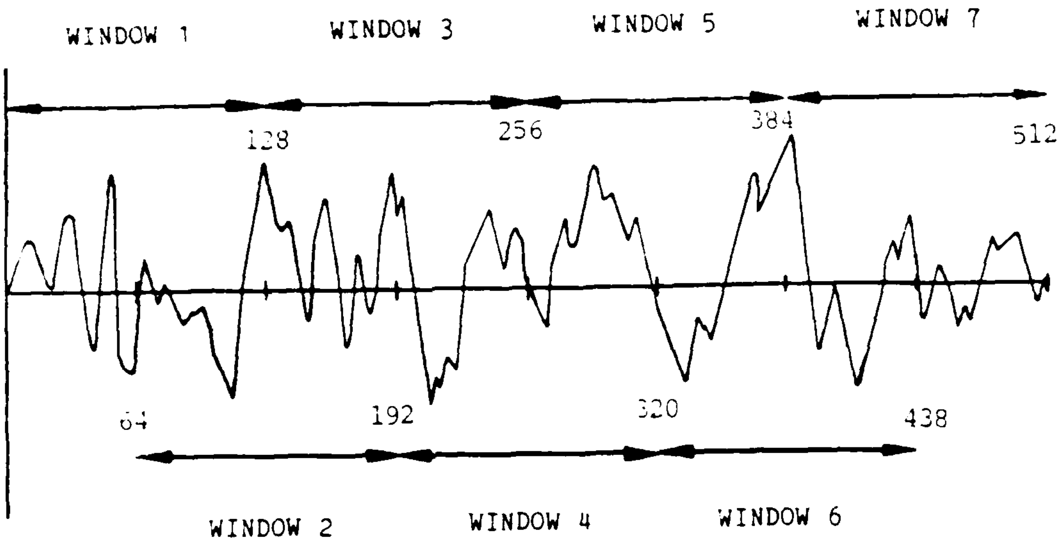


Figure 21 Division of a 512 Point Data String into Seven 128 Point Data Segments

information may not be obtained in some cases until at least five samples per cycle occur. In the case of a 0.05 second sample rate (20 samples per second), the maximum usable frequency of five samples per cycle occurs at just below 21 radians per second. Figure 22 is a plot of the Coherence Function (γ^2) for the data used to generate Figure 20. The Coherence function has been scaled by a factor of 20 therefore frequencies having a gamma squared value of 20 indicate very good correlation between input and output (no noise). Figure 22 shows that above 10 radians/second the gamma squared or coherence function begins to fall significantly indicating poor correlation between input and output (noise). The gamma squared function is determined by:

$$\gamma^2 = \text{Mag } Gxy^2 \;/\; (\text{Mag } Gx * \text{Mag } Gy)$$

Plotting Programs

To make a quick look examination of the input or response time histories, a program called PLOT was written which displays to the Test Pilot School user on the terminal the data string. The plot program is excellent for determining the quality of the time history (excessive noise, low frequency content, wild points, etc)

The frequency response information from the program FRAN (FRequency ANalyzer) is placed into files called BODE.DAT (magnitude and phase information), GX.DAT (power spectral density of input), and GAMMA2.DAT (γ^2 or coherence function). This information is then plotted on semi-log graphs using a modified Test Pilot School plotting routing called BDEPLT. The computer processing for all program is done on the Test Pilot School PDP-11/34 computer. Hard copy plots are produced on a Gould electro-static plotter.

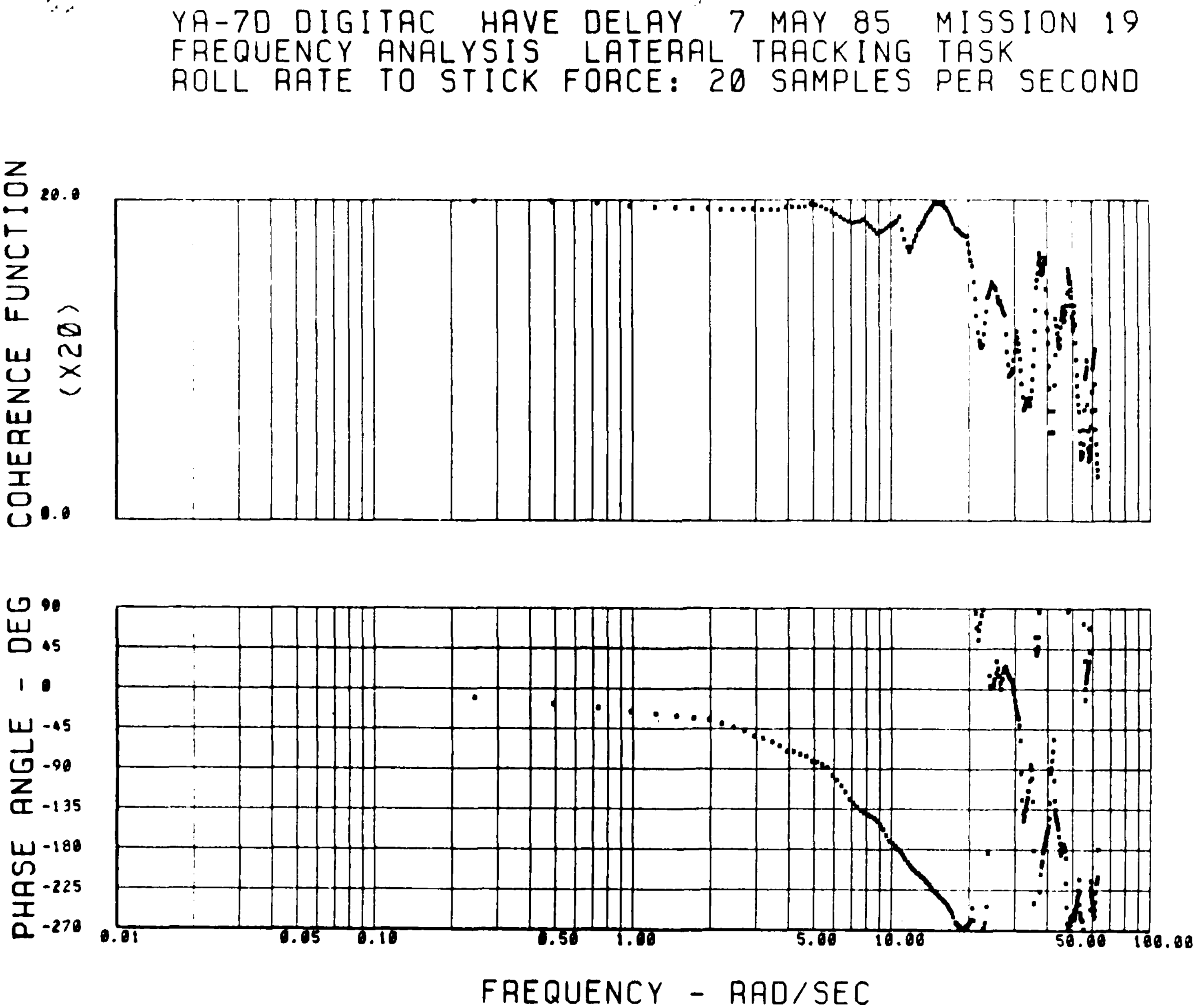


Figure 22 Gamma Squared Function for Configuration HD-075-020

SIMULATED FLIGHT TEST DATA

To verify the FRAN program which performs the frequency analysis, it was desired to test the program not with actual flight test data but rather with simulated data. Several programs were used to generate the simulated flight test data. Program F410 is for an F-4 at 55,000 feet and mach number of 1.8 M. Program F45 is for an F-4 at 35,000, 0.6M, and F89 is for an F89 at 20,000 feet 0.7M. The programs are all identical except for the respective A and B matrices. Each program solves the basic equation:

$$\dot{\underline{x}}(t) = A\underline{x}(t) + Bu(t)$$

where $\underline{x}$ = state vector

u = control vector

a solution of the above is expressed as:

$$x(t) = e^{At}x(0) + \int_0^t e^{A(t-\tau)}Bu(\tau)d\tau$$

If $t = kT$ (discrete time intervals)

$$x(kT) = e^{AkT}x(0) + \int_0^{kT} e^{A(kT-\tau)}\,Bu(\tau)d\tau$$

for the case of k=2,

$$x(2T) = e^{A2T}x(0) + \int_0^{T} e^{A(2T-\tau)}Bu(0)d\tau + \int_0^{2T} e^{A(2T-\tau)}Bu(T)d\tau$$

it can be shown that

$$e^{AkT}x(0) = e^{AT}x[(k-1)T]$$

so in general and using the identity matrix I:

$$x[(k+1)T] = e^{AT}x(kT) + A^{-1}(e^{AT} - I)Bu(kT)$$

e^{AT} was replaced by a series expansion and approximated by the first four terms:

$$e^{AT} = [I + AT + A^2T^2/2! + A^3T^3/3!]$$

giving;

$$X[(k+1)T] = [\sum_{k=0}^{\infty} A^kT^k/k!]x(kT) + [\sum_{k=0}^{\infty} A^kT^{k+1}/(k+1)!]u(kT)$$

which was the basic equation solved in all the programs which generated the simulated flight test data. The program source code is Fortran and the program F410 is shown in Appendix B. Figure 23 shows the flowchart for generating the simulated flight test data.

PROGRAM FEATURES

The program requests the amount of output (time) from the user. The program also requests the sampling rate (i.e. 0.125 seconds per sample). Typically output time and sampling rates were chosen to provide a convenient amount of data for the frequency response program (FRAN) (i.e. 512 or 1024 data points).

While the program generates output at the sample rate requested by the user, it's step size, T, between output can be made smaller at the discretion of the user. The value of T was typically 0.01 seconds. The program was designed with A and B matrices representing the pitch axis. The output of this program was the change from equilibrim of pitch rate (q), pitch angle (θ), velocity (u), and angle of attack, (α). An example of the output is shown in Figure 24. Data files

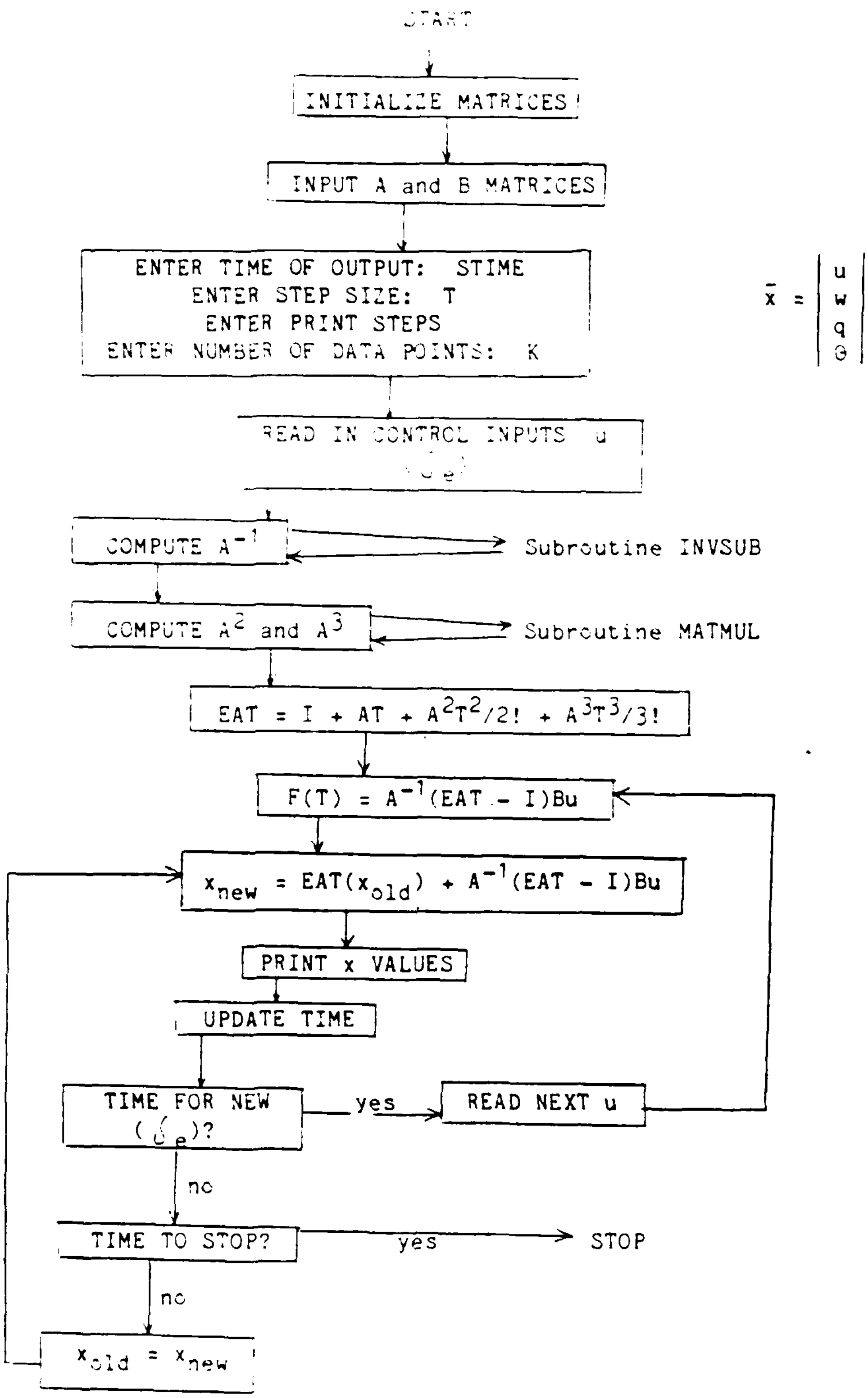


Figure 23 Simplified Flowchart for Programs Generating Simulated Flight Test Data

ENTER NAME OF INPUT DATA: NNNNNN.DAT

VALUES			VELOCITY	ALPHA	Q RATE (D)	THETA DEG
TIME	=	0.12	0.00000	0.00000	0.00000	0.00000
TIME	=	0.25	0.00000	0.00000	0.00000	0.00000
TIME	=	0.37	0.00000	0.00000	0.00000	0.00000
TIME	=	0.52	0.00000	0.00000	0.00000	0.00000
TIME	=	0.62	1.83357E-04	-1.59432E-05	-2.91174E-0[illegible]	-1.93057E-04
TIME	=	0.75	7.57990E-04	-3.05940E-04	-7.08418E-03	-8.42575E-04
TIME	=	0.87	1.59563E-02	-6.54905E-03	-0.22819	-1.69207E-02
TIME	=	1.00	6.95122E-02	-2.79797E-02	-0.67945	-7.68073E-02
TIME	=	1.13	0.17494	-6.71905E-02	-1.1605	-0.19584
TIME	=	1.25	0.31666	-0.11310	-1.3122	-0.35303
TIME	=	1.38	0.46996	-0.15198	-1.2121	-0.51189
TIME	=	1.50	0.61957	-0.17774	-1.0128	-0.65099
TIME	=	1.63	0.75969	-0.19044	-0.80683	-0.76421
TIME	=	1.75	0.89020	-0.19305	-0.63980	-0.85388
TIME	=	1.87	1.0146	-0.18955	-0.53729	-0.92679
TIME	=	2.00	1.1364	-0.18322	-0.47852	-0.98972
TIME	=	2.13	1.2403	-0.16863	-0.17320	-1.0283
TIME	=	2.25	1.2902	-0.13202	0.52414	-1.0014
TIME	=	2.38	1.2456	-6.08384E-02	1.5511	-0.86355
TIME	=	2.50	1.1232	3.01975E-02	2.0124	-0.63478
TIME	=	2.63	0.96921	0.11367	1.9469	-0.38423
TIME	=	2.75	0.81895	0.17372	1.5997	-0.16186
TIME	=	2.88	0.68927	0.20718	1.1875	1.16249E-02
TIME	=	3.00	0.58682	0.21708	0.77836	0.13283
TIME	=	3.12	0.52684	0.20271	0.20605	0.19088
TIME	=	3.25	0.55955	0.14867	-0.96713	0.13517
TIME	=	3.38	0.68759	6.16870E-02	-1.7180	-4.00119E-02
TIME	=	3.50	0.87935	-3.44534E-02	-2.0554	-0.28150
TIME	=	3.63	1.0719	-0.10867	-1.5677	-2.89587E-02
TIME	=	3.75	0.97206	-0.12570	-0.20031	-0.13325
TIME	=	3.88	0.57834	-8.10334E-02	1.0775	-7.01389E-02
TIME	=	4.00	6.40936E-02	-1.84947E-03	1.7595	0.11386
TIME	=	4.13	-0.37698	7.47849E-02	1.6612	0.33030
TIME	=	4.25	-0.51169	0.11352	0.75638	0.47842
TIME	=	4.37	-0.58[illegible]94	0.11112	5.25301E-02	0.52532
TIME	=	4.50	-0.60386	8.49044E-02	0.37820	0.50171
TIME	=	4.63	-0.59316	5.03[illegible]75E-02	-0.55305	0.44121
TIME	=	4.75	-0.27058	[illegible].3524[illegible]E-03	-1.0945	0.33358
TIME	=	4.88	0.24094	-5.81009E-02	1.5356	0.16444
TIME	=	5.00	0.73644	0.11[illegible]95	1.6016	-3.45037E-02

TT10 -- STOP

Figure 24 Output from Program F89 (Program Generating Simulated Flight Test Data) (i.e. VELOCITY.DAT) are created for each of the program's outputs.

PROGRAM FRAN VERIFICATION

To verify the frequency response program (FRAN) the program was run with the simulated flight test data generated from program F410. (F-4 at 55,000 feet and 1.8M). The F-4 at this flight condition is known to have a short period natural frequency (ω_{sp}) of 4.84 rad/sec and a short period damping ratio (ζ_{sp}) of 0.06. The input for the F410 program is elevator deflection (δe). A "random" input signal was created by arbitrarily hand drawing and digitizing an input signal.

Simulating a system which provides output eight times per second, a sampling rate of 0.125 second was entered along with a request for 64 seconds of output. This combination generated 512 data points which is an integer exponential of 2 (a requirement of the FRAN program). The input signal (512 points of δe) along with a simultaneous response signal (512 data points of velocity changes (u), were used as the signals to be frequency analyzed by the program FRAN. The magnitude and phase information created by the program FRAN were plotted. This plot is shown in Figure 25. Note that Figure 25 shows a lightly damped short period to exist near 4.3 rad/sec and the DB increase at this frequency of 10 DB along with the 180 degree phase shift, indicates an equivalent second order damping ratio of about 0.06. This identification of a short period mode by frequency and damping ratio is in very close agreement with the actual values indicating the frequency response program FRAN did in fact work properly. These values were also in close agreement with the eigenvalues determined

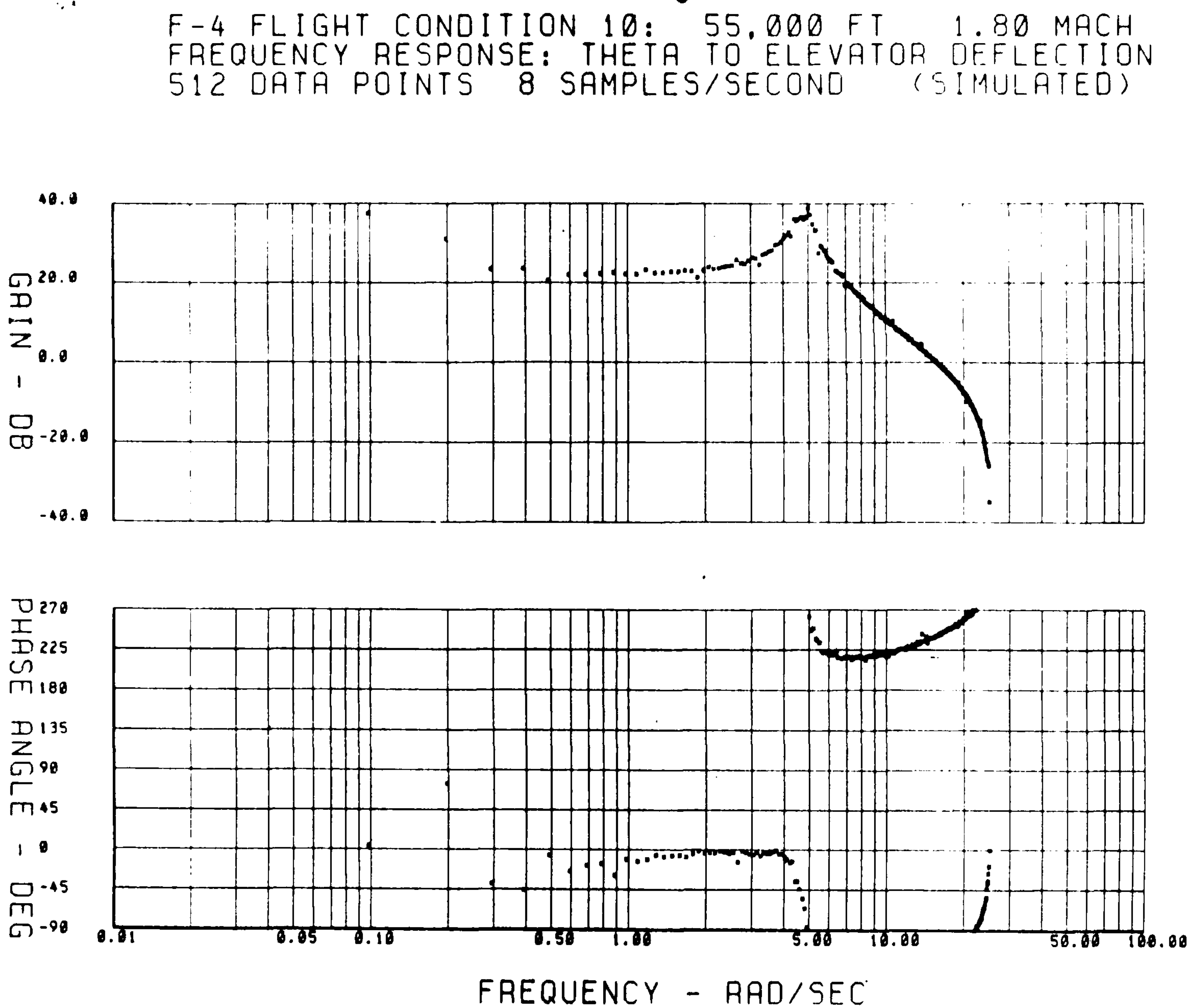


Figure 25 Bode Plot Generated by Frequency Response Program FRAN using Simulated Flight Test Data as Input

from the A matrix which generated the original simulated flight test data. The improper increase in the curve at the higher frequencies (above 10 rad/sec) appears to be a function of poor phase information due to so few sample per cycle and a function by which the response signal is generated in the program F410 (not truly a simultaneous input and response signals). This phenomenon was confirmed by increasing the sampling rate to 20 samples per second. In this case the phase increase was still present but a a much higher frequency-outside the frequency range of interest of 0.1 to 10 radians/second.

V. LOWER ORDER EQUIVALENT SYSTEM DETERMINATION

Once the frequency response plot were determined using the frequency analysis methods of the program FRAN, the next step was to develop a means of reducing the complex, nth order, pilot-in-the-loop aircraft responses to a lower order equivalent system (LOES).

Background

Lower order equivalent systems allow the engineer to compare the responses of a mth-order aircraft to a n-th order aircraft by reducing both systems to a common order. For example, responses of the pitch axis would likely be reduced to 4th order (phugoid and short period modes) or even just second order (short period only). Responses in the roll axis would likely be reduced to a 4th order (spiral, dutch roll, and roll mode) system and may be reduced as low as a first order (roll mode only). Lower order equivalent system also are becoming the basis by which aircraft will be compared to the requirements of the "new" MIL-SPEC (Ref 8). The program which was developed for the United States Air Force Test Pilot School, provides a "best fit" of the actual flight test frequency response data to virtually any transfer function the user may desire. For the pitch axis, the most common LOES used was

$$\frac{\theta}{\delta_e} = \frac{K_\theta \left[s + (1/T_\theta)\right] e^{-T_d s}}{s^2 + 2\zeta_{sp}\omega_{sp} s + \omega_{sp}^2}$$

response and response produced by the LOES. A subroutine calculates a gradient and a gradient search begins to minimize the error. A scaling rule is incorporated to improve search convergence, and a constraint is placed in the search to insure model stability during the search.

The software which was developed for this lower order equivalent system fit utilized subroutines developed at tne Lewis Research Center by Robert C. Seidal (Ref 13). The subroutines were designed to be used with a program which was designed to fit various transfer function models to given experimental frequency response data. Because of the sophistication of the subroutines and gradient calculation, it was determined that use of the subroutines might provide a good means of determining lower order equivalent system fits for flight test data.

Program Formulation

The lower order transfer function provided by the user is matched to actual frequency response over a range of frequencies provided by the user and is typically in the range of 0.1 to 10 rad/sec. The actual frequency response is expressed as a complex number $A(\omega)e^{j\theta(\omega)}$ where A is the magnitude and θ is the phase. The lower order equivalent model transfer function $G(j\omega,b)$, where b is a vector of parameters supplied initially by the user. The cost function J(b) is defined as

$$J(b) = \int_0^{\infty} [\ G(j\omega,b) - A(\omega)\ e^{j\theta(\omega)}\ ]^2\ d\omega \qquad (\omega = \text{frequency})$$

To make the above equation suitable for the computer the following where ω_{sp} = equivalent short period natural frequency

ζ_{sp} = equivalent short period damping ratio

T_d = equivalent time delay

K_0 = gain

For the lateral-directional axis the most common LOES system was

$$\frac{P}{F_s} = \frac{K_p\ (0)\ [\zeta_o, \omega_o]\ e^{-T_d s}}{(1/T_s)(1/\tau_r)[\zeta_{dr}, \omega_{dr}]}$$

where τ_r = equivalent roll mode time constant — P = roll rate

ω_{dr} = equivalent dutch roll natural frequency — F_s = stick force

ζ_{dr} = equivalent dutch roll damping ratio — K = roll rate gain

T_d = equivalent time delay — T_s = spiral time constant

$[\zeta, \omega]$ = damping ratio and natural frequency of quadratic term

For many applications in the lateral axis, such as the DIGITAC HAVE DELAY project (Ref 10), the LOES was simplified to a first order system with time delay which provided good results. The LOES used was;

$$\frac{P}{F_s} = \frac{K\ e^{-T_d s}}{(s + 1/\tau_r)}$$

where K = gain

T_d = equivalent time delay

$1/\tau_r$ = pole location corresponding to the roll mode time constant

The method of determining the best fit of a equivalent system transfer function to frequency response data uses a program which determines a cost function which is the error between actual frequency approximation was used.

$$J(b) = 1/2 \sum_{i=1}^{N_d} [G(j\omega_i, b) - A(\omega_i)e^{j\Theta(\omega_i)}]^2 \; (\omega_{i+1} - \omega_{i-1})$$

where N_d = number of data points.

The choice of the frequencies for the data is important. If the frequencies are improperly chosen, the value of the error at a particular frequency could have a disproportionate effect. In certain cases it is desired to favor a close fit between the model and the plant over a portion of the frequency range. For handling qualities evaluations where this program is most often used, the range of frequencies used was either 0.1 to 10 radians per second or 1 to 10 radians per sec. Probably the best spacing method is to use evenly spaced frequencies from a logrithmic scale.

Lower Order Equivalent System Transfer Function

The transfer function provided by the user as the LOES or the "model", is expressed as a product of a number of different factors.

$$G(s,b) = \prod_{i=1}^{np-nq} g_m(s,b)_i \qquad m = 1,2,3,4,6,8 \tag{9}$$

where $g_1(s,b) = s/b_{1k} + 1$ zero (10)

$g_2(s,b) = b_2$ gain (11)

$g_3(s,b) = (s/b_{3k} + 1)^{-1}$ pole (12)

$g_4(s,b) = [(s^2/b_{5k})^2 + 2sb_4/b_5 + 1]$ quadratic zero (13)

$g_6(s,b) = [s^2/b_{7k} + 2sb_{6k}/b_{7k} + 1]^{-1}$ quadratic pole (14)

$$g_8(s,b) = e^{-sb8} \qquad \text{time delay} \qquad (15)$$

The factors of equation (10), (12), (13), and (14) are repeatable in equation (9). The elements of the b vector are the zeros $b1_k$, the gain b2, the poles $b3_k$, the quadratic zero damping ratios $b4_k$, the quadratic zero natural frequencies $b5_k$, the quadratic pole damping ratios $b6_k$, the quadratic pole natural frequencies $b7_k$, and the lead or lag time b8. The number of $g_m(s,b)$ factors in the transfer function is the number of parameters in the b vector (np) minus the number of quadratic factors (nq).

GRADIENT CALCULATION

The gradient of J(b) is required by the gradient search routine to find the parameters of b which minimize J(b). If parameters are not scaled, many searches will not converge. Gradient search techniques generally require parameter scaling to obtain efficient search convergence. For this program, a scaling law was incorporated such that parameters tend to have equal influence. Each parameter was scaled by its own magnitude. The gradient calculation and scaling routines were provided in a subroutine by NASA. Large changes in the certain values of the b vector during the search may cause a jump across a relatively narrow error boundary. The subroutine prevents values from crossing zero by constraining values as they approach zero. Parameters about to cross zero are assigned a very small value on the same side of zero. This prevents changes between right and left hand plane zeros, and it prevents changes between lead and lag time exponentials.

PROGRAM MECHANIZATION

A complete listing of this program is contained in Appendix B. The main program asks the user if he wishes to use hand entered data or data from a file called FREQ.DAT. Should the user desire to hand enter data, the file FREQ.DAT is automatically created for later use. The program then asks the user to input the LOES transfer function which is to be used for the frequency matching. Up to 15 parameters in the model are available. The user inputs starting values for each of the parameters in the model. Once the LOES transfer function and the starting values have been entered, the user is offered a menu of programming instructions. The menu is: INSTRUCTIONS? 1=SEARCH 2=NEW FREQUENCY DATA 3= NEW TRANSFER FUNCTION 4= PRINT AND SAVE RESULTS 5 = STOP. Instruction 1 causes the search routine to begin and the best fit LOES is determined. Instruction 2 allows the user to re-enter or change frequency data. Hand entered data is inputted by a frequency list, and the corresponding amplitude and phase. Amplitude is given in dB's and phase is in degrees. Instruction 3 reinitializes the program to the point of entering the transfer function. Instruction 4 prints out the frequency response of the LOES after the best fit has been accomplished and also stores the results in a file called "RESULTS.DAT". Instruction 5 stops the program.

The program output contains the value of the cost function (F), the number of times the cost function was calculated, (KNT), the final parameter step size (size), and the program also contains information as to whether or not the search converged or encountered an error.

Chapter VI contains examples of the use of the program LOES with the Frequency Response Data generated from flight test.

VI. RESULTS OF FLIGHT TEST

Introduction

This chapter presents the partial results of a flight test project which incorporated the frequency response methods and Lower Order Equivalent System fit methods presented in this paper.

A project called HAVE DELAY (Ref 10), was conducted by selected members of USAF Test Pilot School Class 84-B. The objective of the Have Delay Project was to investigate the combined effects of equivalent time delay and roll mode time constant on the lateral handling qualities of fighter aircraft. The test aircraft was a YA-7D DIGITAC which was a modified A-7D. The major modification to the DIGITAC aircraft from the basic A-7D is that the analog control augmentation system (CAS) was replaced by a digital control augmentation system (DCAS). The aircraft was also modified with a Yaw, Angle of Attack, Pilot-Statics System (YAPS) head mounted on a flight test nose boom, and sensitive flight test instruments. The testing was conducted in the cruise configuration with 6 MAU-12B/A pylon racks, an instrumentation pod mounted on the right inboard station and an Inert MK-82 mounted on the left inboard station to maintain weight and drag symmetry.

The test was conducted at 15,000 feet (pressure altitude) and involved tracking a target aircraft which was maneuvering at 2.5 g and 350 KIAS. The test was conducted in accordance with the limitation specified in the HAVE DELAY Test Plan, the A-7D Flight Manual, and AFFTC Regulation 55-2 (Ref 11, 2, and 1).

Twenty test sorties totaling 30.5 hours were flown between 2

April 1985 and 8 May 1985 at the Air Force Flight Test Center (AFFTC),. Edwards AFB, California.

The test objectives were designed to evaluate fighter type aircraft. Specifically the objectives were:

1. Correlate lateral handling qualities ratings (Cooper-Harper) to various changes in equivalent time delay in the lateral axis.
2. Correlate pilot handling qualities ratings to changes in equivalent roll mode time constant.
3. Correlate pilot ratings with equivalent bandwidth.

To accomplish the objectives of the Have Delay test program a method of determining the Equivalent Parameters was needed as the Test Pilot School had no means of performing frequency analysis on flight test data. The method used was to incorporate the frequency response software (FRAN) and the Lower Order Equivalent System fit software (LOES) previously described and develop for this thesis.

Test Article Description

The A-7D is a single engine, single place, transonic light surface attack aircraft manufactured by the Vought Aeronautics Company. The A-7D is powered by the Allision TF41-A-1 non-afterburing turbofan engine. A complete aircraft description is contained in the Flight Manual (Ref 2) and the Partial Flight Manual (Ref 4).

Test Instrumentation and Data Reduction

Flight test data was recorded by an on-board magnetic tape. Flight test data was also displayed real-time on ground based strip charts via a Telemetry system (TM). Stick force and roll rate time histories were examined for noise and wild points and if found suitable were then used as the input and response signals for the frequency response analyzer program (FRAN).

Test Methods and Conditions

An A-7 aircraft was used to provide a maneuvering target for the DIGITAC aircraft to track. The maneuvering performed by the target was a "canned" maneuver and did not vary from test point to test point. Project pilots were given an unknown combination of lateral time delay and roll mode time constant. Each pilot then aggressively tracked the maneuvering target and provided a handling qualities rating for that configuration.

The frequency analyzer program, FRAN, provided a means of examining the frequency content of the pilot's stick force (PSD of Fs) to ensure the target was being aggressively tracked and to ensure the aircraft dynamics were being adequately excited throughout the frequency range of interest (out to 10 rad/sec). Figure 26 shows the power spectral density of the input signal, stick force for two such configurations. The first configuration being tested in Figure 26 is for an equivalent roll mode time constant of 0.25 seconds and a equivalent time delay of 150 ms. The other had a 0.50 second time constant and a time delay of 75 ms.

Figure 27 shows the frequency response information (BODE plot) for roll rate to stick force for configuration HD-000-050. In all, 27 different test configurations were evaluated (i.e. 27 different combination of roll mode time constant and time delay). To analyze

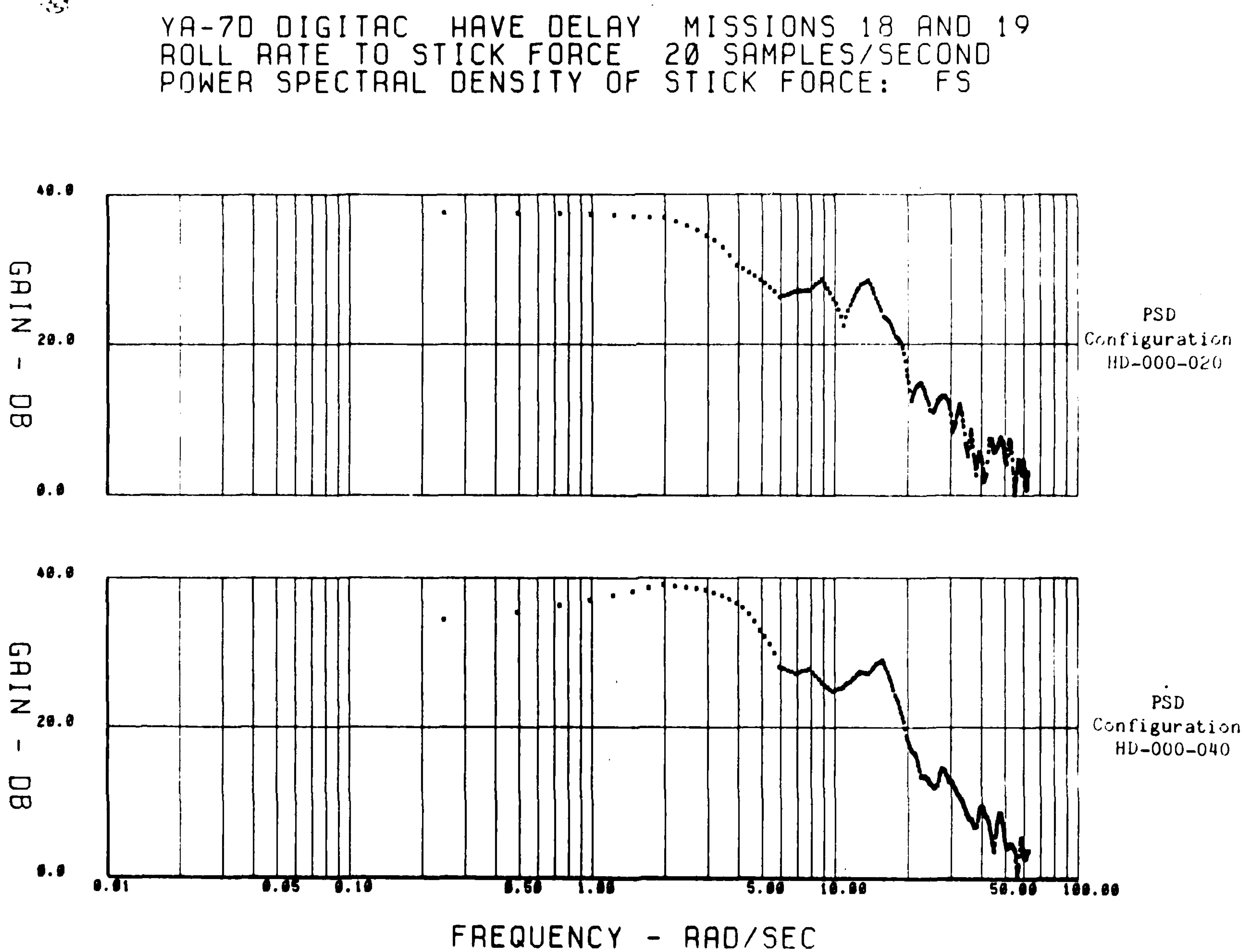


Figure 26 Power Spectral Density of Pilot's Input for Configurations HD-000-020 and HD-000-040

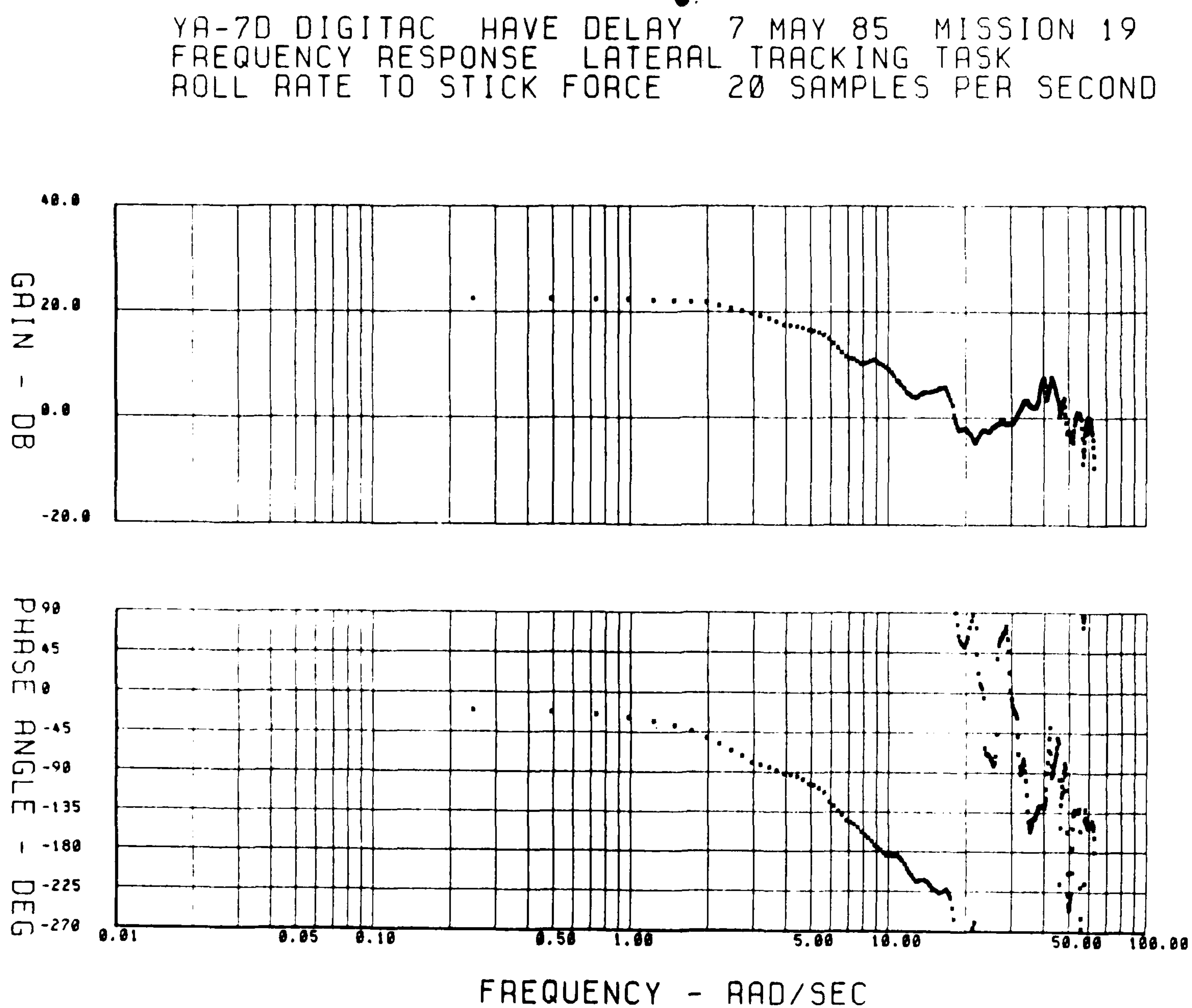


Figure 27 Frequency Response for Have Delay Configuration HD-000-050

the results, 27 different frequency response plots were generated. From the data created to make these plots, the lower order equivalent system fits were made.

Examination of Results

In producing accurate lower order equivalent system fits to actual aircraft response, two main requirements must be met. First the frequency response results must accurately represent the aircraft. Secondly, the LOES fit must sufficiently fit the flight test data. The accuracy of the frequency response methods are dependent on the length of the time history used, data sampling rates, noise reduction techniques, window filter design, and the amount of noise present in the flight test signals. (see chapters 2 and 4).

The accuracy of the LOES fit is simply a matter of examining the aircraft magnitude and phase information and comparing to the magnitude and phase information of the lower order system. The cost function is also an excellent tool for determining the adequacy of the fit. For example in the Have Delay test program, the correlation between pilot ratings and equivalent system parameters appeared to be very poorly defined. However, examination of the LOES fits revealed a number of fits with excessively large cost functions. These poorly fit parameters were removed and then the correlation between pilot ratings and equivalent system parameters was clear and distinct (see Ref 10). The problem appears to be that the assumption that the data be fit with a first order response and time delay for the roll rate to stick force transfer function was not valid in some cases. In fact, due to the manner in which the DIGITAC software was designed, some particular test configuration were significantly higher order and did not reduce well to a first order system. Figure 28 shows and example of both the actual flight data and the LOES data generated when the flight test data was reduce to a first order system with time delay. Figure 29 shows the raw frequency response data and the LOES frequency response data as a comparison.

YA-70 DIGITAC HAVE DELAY 7 MAY 85 MISSION 18
FREQUENCY RESPONSE LATERAL TRACKING TASK
ROLL RATE TO STICK FORCE 20 SAMPLES PER SECOND

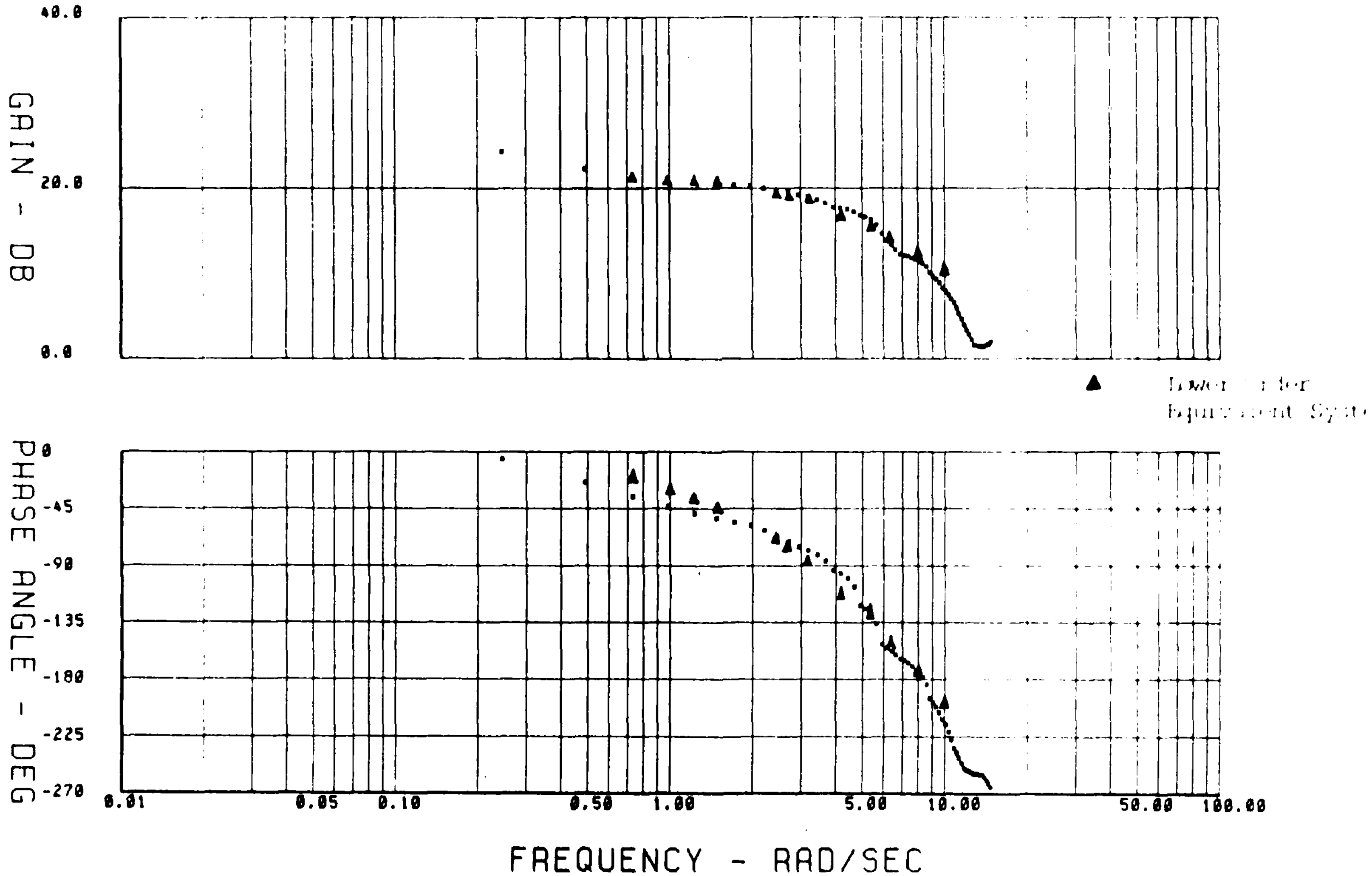


Figure 28 Frequency Response for Configuration HD-075-055 and Lower Order Equivalent System Fit (LOES)

frequency (rad/sec)	amplitude (dB)	phase (deg)
0.73630	21.45400	-35.61970
0.98200	20.99910	-43.55120
1.22700	20.70940	-49.00160
1.47600	20.50880	-52.91250
2.20900	19.98620	-62.20370
2.45400	19.72240	-66.52180
3.19100	18.97010	-78.11060
4.17200	17.64070	-96.30300
5.15400	16.61700	-124.22000
6.13600	13.95190	-155.02000
8.09900	11.44260	-174.55200
10.06000	7.91984	-216.28200

Frequency Response Data for Configuration HD-075-055

frequency (rad/sec)	amplitude (dB)	phase (deg)
0.73630	21.26440	-23.08800
0.98200	21.08311	-30.54047
1.22700	20.86127	-37.77857
1.47600	20.60031	-44.91162
2.20900	19.68699	-64.50171
2.45400	19.35189	-70.58494
3.19100	18.31239	-87.62411
4.17200	16.94656	-107.89897
5.15400	15.66860	-126.20632
6.13600	14.50053	-143.14619
8.09900	12.47462	-174.43457
10.06000	10.78648	-203.66895

Frequency Response Data for Lower Order Equivalent System

$$\text{LOES} = \frac{K\, e^{-T_d s}}{(s/(1/\tau_r) + 1)}$$

τ_r = equivalent roll mode time constant

T_d = equivalent time delay

Cost Function = 27.55664
Gain (K) = 11.8972
$1/\tau_r$ = 3.0598
T_d = 226 ms

Figure 29 Frequency Response Data and Data for Corresponding Lower Order Equivalent System Fit (LOES)

VII. RESULTS, CONCLUSIONS, AND RECOMMENDATIONS

Results

A new and effective means of analyzing aircraft motion has been introduced to the USAF Test Pilot School. These methods have been used by selected members of the TPS during the Have Delay Test Project (Ref 10). Aircraft motion and aircraft flight control systems can now be examined at the Test Pilot School by using frequency response techniques using a program called FRAN. In addition, complex aircraft systems which have been analyzed by these techniques can now be reduced to Lower Order Equivalent Systems using a program called LOES. The use of frequency response methods for pilot-in-the-loop analysis and the use of lower order equivalent systems provide the USAF Test Pilot School with powerful analysis tools.

The main result of this thesis effort is the Test Pilot School student now has the resources available to analyze actual aircraft response in the frequency domain. Time histories of control inputs and aircraft responses can be processed to produce Bode Plots such as shown in Figure 28. Matching of actual frequency response data with Lower Order Equivalent Systems is now available to the Test Pilot School for a larger selection of Lower Order Transfer Functions. The Have Delay project alone utilized the software generated for this thesis for 27 different configurations, generating 27 seperate sets of magnitude and phase plots, power spectral densities plots, and Lower Order Equivalent System fits.

Problems

One problem associated with the frequency response technique is that the method is not designed to handle non-linearities. Aerodynamic non-linearities are kept to a minimum if the task is designed requiring only small pertubations in angle of attack, load factor, etc. Non-linearities would occur if the pilot re-trimmed the aircraft during the maneuver thus invalidating the stick force time history. The flight control system can be a large source of non-linearities. For example if a stick controller contains a breakpoint with significantly different gradients on each side of the breakpoint, non-linear effects will corrupt the analysis. Parabolic stick force shaping would also add a non-linear contribution. Non-linear effects in flight controls are not uncommon and frequency analysis methods which attempt to account for these non-linear effects need to be examined.

It appears that the majority of the uncertainty in the frequency response plots generated by program FRAN can be attributed to the complete lack of signal processing or filtering prior to the signal being digitized. Gardenshirt, Williams, and others (Ref 15), stress the importance of properly processing and filtering flight test data prior to use. Passing the data through a reliable low pass filter prior to digitizing can reduce a portion of the aliasing errors. If the original waveform was not sampled frequently enough, then the waveform may be reconstructed or interpolated after digitizing by a number of reconstruction techniques, either linear or higher order interpolation. Interpolation can also introduce a number of errors of omission from the original waveform. Methods of using higher sample ra[illegible]ased upon reconstructed waveforms and use of anti-aliasing filters are not in use by the USAF Test Pilot School. These techniques need to be examined as a means of improving results of frequency response programs (FRAN).

The manner in which the lower order equivalent system fit is applied to the frequency response data has been shown to be an accurate and reliable manner of determining equivalent system parameters. However improvements are still warranted. The Proposed Mil Handbook (Ref 8) suggests that one may wish to use a slightly different cost function and may even want to use different transfer functions for the LOES fits. The industry needs to standardize the cost functions, frequency range, and transfer functions to be used for frequency response methods.

While much improvement in noise reduction was achieved with the use of averaging overlapping windows and the use of faster sampling rates, the frequency response produced using the programs developed for the Test Pilot School are still too noisy for analysis at the higher frequencies (10-20 rad/sec). Analysis of flight controls, especially, require accurate information past 10 radians per second which is not currently available. Additional studies should be made into ways of decreasing noise and aliasing effects and thus increasing the reliablity of the Test Pilot School new frequency analysis program.

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APPENDIX A

FORTRAN SOURCE CODE LISTING

PROGRAM FRAN

```
      PROGRAM FRAN
C ***********************************************
C  THIS PROGRAM WAS DEVELOPED TO FULLFILL THE
C  REQUIREMENTS OF A MASTERS THESIS BY ALLAN T. REED
C  DEVELOPED AT THE AIR FORCE INSTITUTE OF TECHNOLOGY
C  FROM 1984-85  PROGRAM USES UP TO 512 DATA POINTS
C  FROM TWO SIMULTANEOUS PROCESSES  NO DATA PROCESSING
C  (I.E. NOISE REDUCTION OR FILTERING) IS PERFORMED
C  BY THIS PROGRAM.  DATA PROCESSING MUST BE DONE PRIOR
C  PROGRAM DIVIDES RAW DATA INTO SEPARATE 'WINDOWS' FOR
C  PROCESSING THEN PERFORMS FFT ON THE WINDOWED DATA
C  FREQUENCY INFORMATION IS GENERATED FOR EACH WINDOW
C  AND THEN ALL OF THE WINDOWED DATA IS AVERAGED
C  TOGETHER TO MINIMINIMIZE THE EFFECTS OF NOISE OR ALAISING
C  PROGRAM OPTIMIZED BY LONG TIME HISTORIES AND FAST
C  SAMPLE RATES (20 SAMPLES/SEC MINIMUM)
C ***********************************************
C  A&B ARE INPUT COEFFICIENTS  C&D ARE OUPUT COEFFICIENTS
      DIMENSION A(512),B(512),D(512),R(512),GX(512),W(512)
      DIMENSION GY(512), GXY(512),C(512),S(512)
      DIMENSION AT(1024),CT(1024),PA(512),QA(512)
      BYTE FILE(12)
      DATA FILE/8*0,'.','D','A','T'/
      WRITE (5,100)
100   FORMAT (' ENTER NUMBER OF DATA POINTS AND POWER OF TWO ')
      READ(5,101)N,NU
101   FORMAT (I4)
      PI=3.14159265358
C  ******************************************
C    INITIALIZATION FOR THE WINDOW SIZE
      ND2=N/2
      ND4=N/4
      ND8=N/8
      NUM2=NU-2
C
      WRITE (5,2700)
2700  FORMAT(' ENTER DATA SAMPLE RATE IN SECONDS, I.E. .05')
      READ (5,102) SR
C
      DO 1 L=1,ND2
      GX(L)=0.0
      GXY(L)=0.0
      PA(L)=0.0
      QA(L)=0.0
      GY(L)=0.0
C  W(L) IS THE ARRAY OF FREQUENCIES
      W(L)=(L*2.0*PI)/(FLOAT(N)*SR)
      W(L)=ALOG10(W(L))
1     CONTINUE
C  PROGRAM USES A MINIMUM 50% OVERLAPPING WINDOW FOR FFT CALCULATIONS
      WMIN=0.01
      WMAX=100.
      WRITE(5,200)
200   FORMAT(' ENTER NAME OF INPUT DATA:  NNNNN.DAT ')
      ACCEPT 52, (FILE(I),I=1,8)
52    FORMAT(8A1)
      OPEN(UNIT=1,NAME=FILE,TYPE='UNKNOWN')
      WRITE(5,40)
40    FORMAT(' READING IN INPUT DATA ',/)
      DO 41 I=1,N
C    ARRAY AT(I) IS A FILE FOR ENTIRE INPUT DATA ARRAY
      READ(1,102)AT(I)
41    CONTINUE
      WRITE(5,201)
```

```
201   FORMAT(' ENTER NAME OF RESPONSE DATA:  NNNN.DAT ')
      ACCEPT 52,(FILE(I),I=1,8)
      CLOSE (UNIT=1)
      OPEN (UNIT=2,NAME=FILE,TYPE='UNKNOWN')
      WRITE(5,241)
241   FORMAT(' READING RESPONSE DATA' ,/)
      DO 42 I=1,N
C    ARRAY CT(I) IS A FILE FOR ENTIRE OUTPUT DATA ARRAY
      READ(2,102)CT(I)
42    CONTINUE
C    ****************************************
C
      IF(N.EQ.512) KWNDOW=7
      IF(N.EQ.1024) KWNDOW=15
C    A TOTAL OF 7 OR 15 WINDOWS WILL BE AVERAGED (50% OVERLAP)
      DO 2 K=1,KWNDOW
      WRITE(5,50)
50    FORMAT('  WINDOWING DATA PLUS BLACKMAN FILTERING ',/)
C   ASSIGN INPUT AND OUTPUT DATA FOR EACH FFT WINDOW
C       J = START POINT OF CURRENT WINDOW
C       J1 = STOP POINT OF CURRENT WINDOW
C
      J=((K-1)*64) + 1
      J1=J+128-1
      M=0
C    LOAD APPROPRIATE AMOUNT OF POINTS AND BLACKMAN FILTER
C      BLACKMAN FILTERING IS SIMILAR TO COSINE OR HANN FILTERING
C
      DO 3 I=J,J1
      M=M+1
      R(M)= AT(I)
      S(M)= CT(I)
      ARG=FLOAT(M)/FLOAT(128)
      R(M)=R(M)*(.42-.50*COS(2.*PI*ARG)+.08*COS(4.*PI*ARG))
      S(M)=S(M)*(.42-.50*COS(2.*PI*ARG)+.08*COS(4.*PI*ARG))
3     CONTINUE
102   FORMAT (F15.9)
      WRITE(5,53)
53    FORMAT('  COMPUTING FFT ON INPUT AND OUTPUT DATA',/)
      CALL FFT(R,S,128,7,1)
C
C    FFT CONVERSION COMPLETE
C
C   **************************************
C
C     SEPERATE FFT DATA INTO REAL AND IMAGINARY COMPONENTS OF
C     INPUT AND OUTPUT SIGNALS.
C
      DO 4 I=1,64
      L=I+1
      NMI=128-I+1
      A(I)=(R(L)+R(NMI))/2.
      B(I)=(S(L)-S(NMI))/2.
      C(I)=(S(L)+S(NMI))/2.
      D(I)=-(R(L)-R(NMI))/2.
4     CONTINUE
C
      WRITE(5,104)J,J1
104   FORMAT(' COMPUTING PARAMETERS FOR WINDOW VALUES',I4,' TO',I4,/)
      DO 5 I=1,64
      P=A(I)*C(I) + B(I)*D(I)
      Q=A(I)*D(I) - B(I)*C(I)
C   COMPUTE CROSS SPECTRAL DENSITY OF INPUT TO OUTPUT.  GXY
      GXY(I)= GXY(I)+SQRT(P**2. + Q**2.)
C   COMPUTE POWER SPECTRAL DENSITY OF INPUT.  GX
```

```
      GX(I)=GX(I)+A(I)**2. + B(I)**2.
      PA(I)=PA(I) + P
      QA(I)=QA(I) + Q
C  COMPUTE GY = PSD OF OUTPUT SIGNAL,  GY
      GY(I)=GY(I)+C(I)**2. + D(I)**2.
5     CONTINUE
2     CONTINUE
C
C  ****************************************
C
      CLOSE(UNIT=2)
C  AVERAGE ALL VALUES
      WRITE(5,60)
60    FORMAT(' AVERAGING PARAMETERS AND INTERPOLATING FREQUENCIES',/)
      MID =64
      LAST=128
      NTL=LAST-1
      F=FLOAT(KWNDOW)
      DO 6 I=1,MID
      GXY(I)=GXY(I)/F
      GX(I)=GX(I)/F
      GY(I)=GY(I)/F
      PA(I)=PA(I)/F
      QA(I)=QA(I)/F
6     CONTINUE
C  ASSIGN TEMPORARY VALUES TO FILL OUT FREQUENCY RANGE
98    DO 66 I=1,MID
      A(I)=GXY(I)
      B(I)=GX(I)
      C(I)=GY(I)
      AT(I)=PA(I)
      CT(I)=QA(I)
66    CONTINUE
C  FILL OUT ENTIRE FREQUENCY RANGE
      DO 7 I=2,LAST,2
      ID2=I/2
      GXY(I)=A(ID2)
      GX(I)=B(ID2)
      GY(I)=C(ID2)
      PA(I)=AT(ID2)
      QA(I)=CT(ID2)
7     CONTINUE
C  INTERPOLATE INTERMEDIATE VALUES
      DO 8 I=3,NTL,2
      IM1=I-1
      IP1=I+1
      GXY(I)=(GXY(IM1)+GXY(IP1))/2.
      GX(I)=(GX(IM1)+GX(IP1))/2.
      GY(I)=(GY(IM1)+GY(IP1))/2.
      PA(I)=(PA(IM1)+PA(IP1))/2.
      QA(I)=(QA(IM1)+QA(IP1))/2.
8     CONTINUE
C  LINEARLY INTERPOLATE FIRST AND LAST VALUES OF RECORDS
      GXY(1)=(GXY(2)-GXY(3))+GXY(2)
      GX(1)=(GX(2)-GX(3))+GX(2)
      PA(1)=(PA(2)-PA(3))+PA(2)
      QA(1)=(QA(2)-QA(3))+QA(2)
      GY(1)=(GY(2)-GY(3))+GY(2)
      NN=NTL-1
      GXY(LAST)=(GXY(NTL)-GXY(NN))+GXY(NTL)
      GX(LAST)=(GX(NTL)-GX(NN))+GX(NTL)
      PA(LAST)=(PA(NTL)-PA(NN))+PA(NTL)
      QA(LAST)=(QA(NTL)-QA(NN))+QA(NTL)
      GY(LAST)=(GY(NTL)-GY(NN))+GY(NTL)
C  IF ND2 DATA POINTS, FINISHED: IF NOT, REPEAT DOUBLING ROUTINE
```

```
      IF ( MID .EQ. ND4) GO TO 99
      LAST = LAST*2
      NTL = LAST-1
      MID = MID*2
      GO TO 98
99    CONTINUE
C  COMPUTE HXY=GXY/GX AND GAMMA SQUARED VALUES
      OPEN(UNIT=1,NAME='BODE.BDE',TYPE='UNKNOWN')
C  CONVERT HXY TO DECIBLES
C  LET A(I) BE THE HXY(I) VALUES
C  LET B(I) BE THE GAMMA SQUARED, GAMMA2(I) VALUES
C  LET C(I) BE THE PHASE(I) VALUES
      DO 9 I=1,ND2
      A(I)=GXY(I)/GX(I)
      A(I)=20.*ALOG10(A(I))
      B(I)=20.*(GXY(I)**2.)/(GX(I)*GY(I))
      C(I)=ATAN2(QA(I),PA(I))*57.29578
      IF(C(I) .GT. 90.) C(I)=C(I)-360.
C   CONVERT GX AND GY TO POWER SPECTRAL DENSITIES
      GX(I)=10.*ALOG10(GX(I))
      GY(I)=10.*ALOG10(GY(I))
9     CONTINUE
      OPEN(UNIT=2,NAME='GAMMA2.BDE',TYPE='UNKNOWN')
      WRITE(2,13600)ND2,WMIN,WMAX
13600 FORMAT(X,I3,G16.6,G16.6)
      DO 10 I=1,ND2
      WRITE(2,13700)B(I),C(I),W(I)
13700 FORMAT(X,3G16.6)
10    CONTINUE
      CLOSE (UNIT=2)
C  WRITE REMAINING DATA TO DISK
      OPEN(UNIT=2,NAME='GX.BDE',TYPE='UNKNOWN')
      WRITE(2,13600)ND2,WMIN,WMAX
      WRITE(1,13600)ND2,WMIN,WMAX
      DO 13 I=1,ND2
      WRITE(1,13700) A(I), C(I), W(I)
      WRITE(2,13700) GX(I), C(I), W(I)
13    CONTINUE
      STOP
      END
```

APPENDIX B

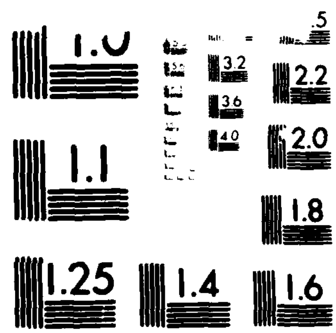
FORTRAN SOURCE CODE LISTING

PROGRAM LOES

```
C     PROGRAM LCES FOR SYSTEM IDENTIFICATION BY MATCHING TRANSFER
C     FUNCTION TO GIVEN SET OF FREQUENCY DATA.  TRANSFER FUNCTION
C     IS SELECTED BY USER THEN PROGRAM PERFORMS CONJUGATE GRADIENT
C     SEARCH TO OPTIMIZE COST FUNCTION.  PROGRAM REQUIRES TWO SUB-
C     ROUTINES:   CFG  AND  CGFM    THESE SUBROUTINES PERFORM THE
C     COST FUNCTION CALCULATIONS AND THE GRADIENT SEARCH.
      PROGRAM LOES
      DIMENSION B(15),G(15),ID(15),AMP(50),PHA(50),W(51),H(30)
      DIMENSION WR(50),DB(50)
      COMPLEX PLANT(50)
      EXTERNAL CFG
      LOGICAL KG
C    .TRUE. KG MEANS TO COMPUTE GRADIENT
      COMMON/IDTN/PLANT,W,ID,K1,INST,ND,KNT,NP,WR
      COMMON/FMC/KOUNT,KG
C   **************************************************
C     PROGRAM VARIABLE LIST
C
C     AMP   SYSTEM FREQUENCY RESPONSE MAGNITUDE (VECTOR)
C     B     MODEL PARAMETERS (INPUT VARIABLE VECTOR)
C     CFG   SUBROUTINE WHICH COMPUTES COST FUNCTION AND GRADIENT
C     EPS   PARAMETER CHANGE DEFINING SEARCH CONVERGENCE   10E-5
C     F     COST FUNCTION    F=J(B)
C     G     COST FUNCTION SCALED GRADIENT (VECTOR)
C     H     STORAGE VECTOR
C     ID    INTEGER VECTOR THAT IDENTIFIES CORRESPONDING B PARAMETER
C     IER   SEARCH CONVERGENCE PARAMETER (0=CONVERGED: SIZE<EPS)
C     INST   BRANCHING INSTRUCTIONS
C     ITER   SET EQUAL TO LIMIT(200) UNLESS INST = 4 OR 5
C     J      INDEX OF ELEMENT IN VECTOR
C     JD    NUMBER OF NONITERATIVE CONSTANTS IN MODEL
C     KG    LOGICAL VARIABLE   .TRUE. MEANS COMPUTE GRADIENT
C     KNT   COUNT OF COST FUNCTION EVALUATIONS
C     KOUNT   COUNT OF LINE SEARCH ITERATIONS
C     K1 EXPONENT OF FREE S IN MODEL (INPUT VARIABLE)
C     LIMIT    MAXIMUM NUMBER OF ITERATIONS (SET = 200)
C     N      NUMBER OF ITERATIVE PARAMETER OF B VECTOR
C     ND     NUMBER OF FREQUENCY POINTS OVER WHICH INTEGRATION IS DONE
C     NP     TOTAL NUMBER ITERATIVE PLUS CONSTANT MODEL PARAMETERS
C     PHA   SYSTEM FREQUENCY RESPONSE PHASE (DEG) (INPUT VECTOR)
C     PLANT   CONVERTED SYSTEM RESPONSE TO EQUIVALENT COMPLEX NUMBER
C     SIZE    PARAMETER STEP SIZE  (SET TO 0.1 AT START OF SEARCH)
C     THETA   SCRATCH VARIABLE FOR RADIAN PHASE
C     W      RADIAN FREQUENCY VECTOR  (RADIANS/SECOND)
C
C   ************************************************************
      PI=3.1415926535
10    WRITE (5,110)
110   FORMAT(' ENTER NUMBER OF FREQUENCY DATA POINTS FOR CURVE FIT',/)
      READ (5,111)ND
111   FORMAT (I5)
      WRITE(5,112)
112   FORMAT(' FREQUENCY DATA? 1=HAND ENTER  2=FROM FILE FREQ.DAT ',//)
      READ(5,140) KK
      IF(KK .EQ. 2) GO TO 15
      WRITE(5,103)
103   FORMAT(' ENTER ALL FREQUENCIES IN ORDER (RAD/SEC) ',//)
      DO 2 I=1,ND
      READ (5,113) W(I)
  2   CONTINUE
      WRITE (5,114)
114   FORMAT(' ENTER CORRESPONDING AMPLITUDES (DECIBLES) ',//)
      DO 3 I=1,ND
      READ (5,113) DB(I)
```

AD-A168 604 EVALUATION OF A FREQUENCY RESPONSE TECHNIQUE FOR AIRCRAFT SYSTEM IDENTIFICATION(U) AIR FORCE INST OF TECH WRIGHT-PATTERSON AFB OH SCHOOL OF ENGI.. A T REED 2/2

UNCLASSIFIED 31 OCT 85 AFIT/GAE/AA/85J-2 F/G 17/7 NL


1.0
1.1
1.25
1.4
1.6
1.8
2.0
2.2
3.2
3.6
4.0

```
C  ***********************************************
C       WRITE (5,183)
183     FORMAT (' ENTER MAX NUMBER OF ITERIATIONS DESIRED',//)
C       READ (5,111) LIMIT
C       WRITE (5,184)
184     FORMAT (' ENTER VALUE OF EXPONENT OF THE FREE S ',//)
C       READ (5,111) K1
C       WRITE (5,185)
185     FORMAT (' PROGRAM WILL ITERATE ON ALL B VALUES UNLESS YOU ',/,
     1' CHOOSE THE LAST ENTERED B VALUES TO BE FIXED. ',/,
     2'     ENTER NUMBER OF FIXED B VALUES (NORMALLY = 0)   ',//)
C       READ (5,111) JD
C  ***************************************************
36      N=N-JD
        NP=N+JD
C  NP IS THE TOTAL NUMBER OF TERMS BOTH ITERATIVE AND CONSTANTS
        WRITE (5,130) LIMIT,N,K1,JD,(ID(J),J=1,NP)
130     FORMAT(' LIMIT,N,K1,JD=',4I3,'    ID=',15I3)
        GO TO 87
777     WRITE (5,186)
186     FORMAT(/,' INSTRUCTIONS? 1= SEARCH  2= NEW FREQUENCY DATA ',/,
     1' 3= NEW TRANSFER FUNCTION  4= PRINT & SAVE RESULTS  5=STOP ')
        READ (5,140) INST
140     FORMAT (I1)
        GO TO (40,10,30,50,1000),INST
50      WRITE (5,160)
160     FORMAT(6H MODEL,/,8X,1HW,14X,3HAMP,11X,3HPHA)
40      SIZE=.1
C   SIZE IS THE PARAMETER STEP SIZE
        EPS=1.E-6
C  EPS IS THE PARAMETER CHANGE DEFINING CONVERGENCE
        ITER=LIMIT*(1-INST/4)
C  IF INST = 4 ITER BECOMES ZERO AND NO ITERATIONS TAKE PLACE
        KNT=0
C  KNT RECORDS NUMBER OF TIMES THE COST FUNCTION IS EVALUATED
        CALL CGFM(CFG,N,B,F,G,SIZE,EPS,ITER,IER,H)
        WRITE(5,150) F,IER,KOUNT,KNT,SIZE
C  F IS THE COST FUNCTION
150     FORMAT(' F=',F15.5,/,
     1' ERROR MSG = (IER) (0=CONVERGE,1=NO CONVERGE,2=ERROR) ',/,
     2' IER=',I2,'  KOUNT,KNT,SIZE=',I3,I4,1PE10.3)
87      CONTINUE
        WRITE(5,151) (B(J),J=1,NP)
151     FORMAT(' B=',8F11.4)
        GO TO 777
1000    STOP
        END
C  ************************************************
C   SUBROUTINE CFG COMPUTES THE COST FUNCTION AND GRADIENT
C
        SUBROUTINE CFG(N,B,F,G)
C
C   CFG COMPUTES THE COST FUNCTION AND GRADIENT
        DIMENSION B(1),G(1),ID(15),W(51)
        COMPLEX PLANT(50),Z(15),S,ST,GM,E
        LOGICAL KG
        COMMON/IDTN/PLANT,W,ID,K1,INST,ND,KNT,NP
        COMMON/FMC/KOUNT,KG
C  ***********************************************
C   SUBROUTINE CFG VARIABLE LIST
C
C   AM    MAGNITUDE OF G(JW,B)
C   DEG   ANGLE OF G(JW,B) IN DEGREES
C   E     ERROR BETWEEN MODEL AND DATA
C   GM    PARTIAL PRODUCT   BECOMES G(S,B)
```

```
C    IDK  VALUE OF ID(K)
C    J     FREQUENCY INDEX
C    K     PARAMETER INDEX
C    S     S
C    ST    TEMPORARY VALUE
C    WS    SAVED W VALUE
C    Z     PARTIAL PRODUCT IN MODEL GRADIENT (VECTOR)
C
C   ************************************************
      IF(INST .EQ. 4) OPEN(UNIT=2,NAME='RESULTS.DAT',TYPE='UNKNOWN')
      KNT=KNT+1
      DO 30 J=1,N
      G(J)=0.
30    CONTINUE
      WS=W(1)
      F=0.
      DO 100 J=1,ND
      S=CMPLX(0.,W(J))
C   S = LAPLACE VARIABLE
      GM=S**K1
      DO 8 K=1,NP
      IDK=ID(K)
      GO TO (1,3,1,2,8,2,8,4),IDK
C   START HERE IF DEALING WITH A ZERO OR A POLE
1     GM=GM*(S/B(K)+1.)**(2-IDK)
C    GM IS A PARTIAL PRODUCT OF TRANSFER FUNCTION
      IF (KG) Z(K)=S/(S+B(K))*(FLOAT(IDK)-2.)
      GO TO 8
C   START HERE IF USING MODEL AS PLANT
2     ST=(S/B(K+1))**2+2.*S*B(K)/B(K+1)+1.
C   ST IS A TEMPORARY VALUE
      GM=GM*ST**(5-IDK)
      IF(.NOT.KG) GO TO 8
      Z(K)=2.*S*B(K)/(B(K+1)*ST)*(5.-FLOAT(IDK))
      Z(K+1)=-Z(K)*1.+S/((B(K)*B(K+1)))
      GO TO 8
C   Z IS THE PARTIAL PRODUCT IN MODEL GRADIENT
C   START HERE IF DEALING WITH THE GAIN
3     GM=GM*B(K)
      Z(K)=CMPLX(1.,0.)
      GO TO 8
C   START HERE IF DEALING WITH TRANSPORT LAG (DEAD TIME)
4     GM=GM*CEXP(-S*B(K))
      Z(K)=-S*B(K)
8     CONTINUE
      E=GM-PLANT(J)
C   E  REPRESENTS THE ERROR BETWEEN THE MODEL AND THE PLANT
      F=F+(W(J+1)-WS)*REAL(E*CONJG(E))
      IF(.NOT. KG) GO TO 100
      ST=(W(J+1)-WS)*E*CONJG(GM)
      DO 80 K=1,N
      G(K)=G(K)+REAL(ST*CONJG(Z(K)))
80    CONTINUE
      IF(INST.EQ.1) GO TO 100
      AM=CABS(GM)
      DB=20.*ALOG10(AM)
C   AM IS THE MAGNITUDE OF THE PARTIAL PRODUCTS
      DEG=ATAN2(AIMAG(GM),REAL(GM))*57.295780
      IF (DEG .GT. 90.) DEG=DEG-360.
      WRITE(2,120) W(J),DB,DEG
      WRITE(5,120) W(J),DB,DEG
120   FORMAT(3F15.5)
      IF(INST.EQ.5) PLANT(J)=GM
100   WS=W(J)
      IF(INST .EQ. 4) WRITE(2,200)F,(B(J),J=1,NP)
```

```
200   FORMAT(' F=',F15.5,/,' B=',8F11.4)
      IF(INST .EQ. 4) CLOSE(UNIT=2)
      RETURN
      END
C ****************************************************
C  SUBROUTINE CGFM CONDUCTS THE CONJUGATE GRADIENT SEARCH
C
      SUBROUTINE CGFM(CFG,N,B,F,G,SIZE,EPS,ITER,IER,H)
C
C    CGFM PERFORMS THE ACTUAL MINIMIZATION OF THE CONJUGATE
C    GRADIENT SEARCH AS STARTED BY SUBROUTINE CFG
      DIMENSION B(1),H(1),G(1)
      LOGICAL KG
      COMMON/FMC/KOUNT,KG
C  ****************************************************
C     SUBROUTINE CGFM VARIABLE LIST
C
C   BETA   CONJUGATE DIRECTION WEIGHTING
C   FS     SAVED F
C   FSS    SAVED FS
C   J      PARAMETER INDEX
C   JN     J/N
C   K      INDICATOR FOR STEP SIZE REDUCTION
C   L      NUMBER STEP SIZE INCREASES WITHIN ITERATION
C   NCYC   NUMBER OF ITERATION BEFORE RESTARTING CONJUGATE SEARCH
C   R1,R2,R3   TERMS IN QUADRATIC CURVE FIT
C   SCALE  STEP SIZE SCALE FACTOR
C   SCASV  SAVED SCALE
C   STEP   STEP SIZE
C   TSAVE  SAVED TSQR
C   TSQR   SQUARED GRADIENT TERMS SUM
C   X1,X2  PARTIAL PRODUCT
C
C  ****************************************************
C  INITIALIZATIONS
      KOUNT=0
      STEP=2.
      IER=-5
5     BETA=0.
C   BETA IS THE WEIGHTING ON THE CONJUGATE DIRECTION
      NCYC=0
C  NCYC IS THE NUMBER OF ITERATIONS BEFORE RESTARTING CONJUGATE SEARCH
15    K=0
      DO 20 J=1,N
      H(J)=B(J)
20    CONTINUE
      KG=.TRUE.
      CALL CFG(N,B,F,G)
C  TEST FOR STOPPING SEARCH
      IF(KOUNT .GE. ITER) IER=1
      IF(SIZE .LT. EPS) IER=0
40    IF(IER .GT. -2) RETURN
C   COMPUTE PAST GRADIENT WEIGHTING
      TSQR=0.
C  TSQR IS THE SUM OF THE SQUARED GRADIENT TERMS
      DO 50 J=1,N
50    TSQR=TSQR+G(J)**2
      IF(NCYC .EQ. 0) GO TO 60
      BETA = TSQR/TSAVE
60    SCALE=0.
      DO 70 J=1,N
      JN=J+N
      H(JN)=-G(J)+BETA*H(JN)
70    SCALE=SCALE+ABS(H(JN))
      IF(SCALE .GT. 0.) GO TO 80
```

```
      IER=0
      GO TO 40
80    SCALE=SIZE/SCALE
      TSAVE=TSQR
      NCYC=NCYC+1
      FSS=F
      L=1
C   L COUNTS NUMBER OF ITERATIONS
C    UPDATE THE VALUES OF THE B'S
100   DO 110 J=1,N
      JN=J+N
110   B(J)=H(J)*ABS(1.+SCALE*H(JN))
      FS=F
      KG=.FALSE.
      CALL CFG(N,B,F,G)
C   LOGIC CHANGES LINE SEARCH STEP SIZE OF CONCLUDES SEARCH
      IF(K.GT.0) GO TO 120
      IF(F.LT.FS) GO TO 130
      IF(L.GT.1) GO TO 140
      GO TO 150
120   IF(F.LT.FSS) GO TO 140
      GO TO 160
130   SCASV = SCALE
C   SCASV IS THE SAVED VALUE OF THE SCALE
      L=L+1
      SCALE=SCALE*STEP
      FSS=FS
      IF(L.LT.15) GO TO 100
      IER =2
C   IER VALUE OF 2 MEANS ERROR HAS OCCURRED
      GO TO 40
C  NOW FIT QUADRATIC CURVE TO 3 PTS. BRACKETING LINE SEARCH MIN.
140   DO 148 J=1,N
      JN=J+N
      R1=H(J)
      R2=H(J)*ABS(1.+SCASV*H(JN))
      R3=B(J)
      IF(L.GT.3) R1=R1+(R2-R1)/STEP
      X1=(FSS-FS)*(R1-R3)
      X2=(FSS-F)*(R2-R1)
      IF(ABS(R2-R3).GT.EPS/4.) GO TO 147
      IF(L.GT.1) B(J)=R2
      GO TO 148
147   B(J)=(X1*(R1+R3)+X2*(R1+R2))/((X1+X2)*2.)
148   IF(B(J)*H(J).LE.0.) B(J)=-1.*B(J)+EPS*R3
C   UPDATE SEARCH VARIABLES
      SIZE=SIZE*(FLOAT(L)+2.)/4.
      KOUNT=KOUNT+1
      IF(NCYC.GT.N) GO TO 5
      GO TO 15
150   SCASV=SCALE
      SCALE=SCALE/(1.+STEP)
      K=K+1
      SIZE=SIZE/(1.+STEP)
      GO TO 100
160   SIZE=SIZE/(1.+STEP)
      DO 180 J=1,N
180   B(J)=H(J)
      GO TO 5
      END
```

APPENDIX C

FORTRAN SOURCE CODE LISTING

PROGRAM F410

```
      PROGRAM F410
C    F4 AT FLIGHT CONDITION NO. 10
      DIMENSION A(4,4), CMAT(4,4),EAT(4,4),EATM1(4,4)
      DIMENSION IM(4,4),AT(4,4),ATT2(4,4),FCAPT(4,4)
      DIMENSION BU(4,4),X(4,4),AI(4,4),ASQ(4,4),DEL(512)
      DOUBLE PRECISION CMAT
      REAL IM
      INTEGER RA,CA
      BYTE FILE(12),OFILE(14)
      DATA FILE/8*0,'.','D','A','T'/
      RA=4
      CA=4
100   FORMAT (I1)
      NUT=0
C   INITIALIZE ALL MATRICIES
      DO 1 I = 1,RA
      DO 2 J = 1,RA
      ASQ(I,J)=0.0
      EAT(I,J)=0.0
      EATM1(I,J)=0.0
      AI(I,J)=0.0
      AT(I,J)=0.0
      X(I,J)=0.0
      ATT2(I,J)=0.0
      FCAPT(I,J)=0.0
      BU(I,J)=0.0
2     IM(I,J) = 0.0
1     CONTINUE
C   SYSTEM A-MATRIX HERE
      A(1,1)=-.00528
      A(1,2)=-.00501
      A(1,3)=-100.3
      A(1,4)=-31.947
      A(2,1)=.000474
      A(2,2)=-.319
      A(2,3)=1738
      A(2,4)=1.842
      A(3,1)=.00175
      A(3,2)=-.0133
      A(3,3)=-.3138
      A(3,4)=0.0
      A(4,1)=0.0
      A(4,2)=0.0
      A(4,3)=1.0
      A(4,4)=0.0
      WRITE (5,13)
13    FORMAT (' ENTER TIME OF OUTPUT, STEP SIZE,AND PRINT STEPS',/)
      READ (5,200) TIME,CAPT
      READ (5,201) IPRNT
201   FORMAT (1I5)
200   FORMAT (1F15.9)
      STIME=0.0
      DTIME = TIME+.1
      B11=3.42
      B21=-20.7
      B31=-4.90
      B41=0.0
      WRITE (5,203)
203   FORMAT (' ENTER NUMBER OF DATA POINTS ',/)
      READ (5,201) K
      WRITE (5,51)
51    FORMAT(' ENTER NAME OF INPUT DAT:  NNNNNN.DAT',/)
      ACCEPT 52, (FILE(I),I=1,8)
52    FORMAT (8A1)
```

```
      OPEN(UNIT=3,NAME=FILE,TYPE='UNKNOWN')
      DO 5 I=1,K
      READ(3,200) DEL(I)
5     CONTINUE
      CLOSE(UNIT=3)
C    OPEN FILES FOR OUTPUT
      OPEN(UNIT=1,NAME='VELOCITY.DAT',TYPE='UNKNOWN')
      OPEN(UNIT=2,NAME='ALPHA.DAT',TYPE='UNKNOWN')
      OPEN(UNIT=3,NAME='Q.DAT',TYPE='UNKNOWN')
      OPEN(UNIT=4,NAME='THETA.DAT',TYPE='UNKNOWN')
C  START BY SETTING AI EQUAL TO A MATRIX
      DO 3 I=1,RA
      DO 4 J=1,CA
4     AI(I,J)=A(I,J)
3     CONTINUE
C  INVERT THE A MATRIX USING INVSUB; A MATRIX INVERSION SUBROUTINE
C
      CALL INVSUB (A,AI,RA,CA)
C
C  MATMUL IS A MATRIX MULTIPLICATION SUBROUTINE
      CALL MATMUL (A,A,CMAT,RA)
      DO 31 I=1,RA
      DO 32 J= 1,CA
32    ASQ(I,J)=CMAT(I,J)
31    CONTINUE
      JJ=0
C   EAT = I + AT + (A**2)T**2 / 2 FACTORIAL + (A**3)T**3 / 3 FACTORIAL
7     CONTINUE
      DO 91 I=1,RA
      DO 92 J=1,CA
      AT (I,J)= A(I,J)*CAPT
      ATT2(I,J)= (ASQ(I,J)*CAPT**2.)/2.
92    EAT(I,J) =  AT(I,J) + ATT2(I,J)
91    CONTINUE
      DO 64 I=1,CA
      EAT (I,I)=EAT (I,I)+1.0
64    CONTINUE
      CALL MATMUL (A,ASQ,CMAT,RA)
      DO 33 I=1,RA
      DO 34 J=1,CA
34    EAT(I,J)=EAT(I,J)+(CMAT(I,J)*CAPT**3.)/6.0
33    CONTINUE
C  FIND F(T) MATRIX FUNCTION
      DO 19 I=1,RA
      DO 29 J=1,CA
29    EATM1(I,J)=EAT(I,J)
19    CONTINUE
      DO 21 I=1,RA
      EATM1(I,I) = EAT(I,I) - 1.0
21    CONTINUE
22    JJ=JJ+1
      BU(1,1)=B11*DEL(JJ)
      BU(2,1)=B21*DEL(JJ)
      BU(3,1)=B31*DEL(JJ)
      CALL MATMUL (EATM1,BU,CMAT,RA)
      DO 23 I = 1,RA
      DO 24 J = 1,CA
24    FCAPT(I,J) = CMAT(I,J)
23    CONTINUE
C
      CALL MATMUL (AI, FCAPT,CMAT,RA)
      DO 25 I = 1,RA
      DO 26 J= 1,CA
26    FCAPT(I,J)=CMAT(I,J)
25    CONTINUE
```

```
77     CONTINUE
C    MAIN ITERATION OCCURS HERE
       CALL MATMUL (EAT,X,CMAT,RA)
       DO 37 I=1,RA
       DO 38 J=1,CA
38     X(I,J) = CMAT(I,J)
37     CONTINUE
998    CONTINUE
       DO 39 I=1,RA
       DO 41 J=1,CA
41     X(I,J)=X(I,J)+FCAPT(I,J)
39     CONTINUE
       IF(STIME .EQ. 0.0) WRITE(5,11)
11     FORMAT(//,' DELTA VALUES',5X,'VELOCITY',8X,'ALPHA',8X,
     1'P RATE (Q)',7X,'THETA DEG',/)
       STIME=STIME + CAPT
       IF ( STIME .GE. DTIME ) STOP
       NUT=NUT +1
       IF (NUT .EQ. IPRNT) GO TO 3999
       GO TO 77
3999   TIME=STIME
       NUT = 0
       CMAT(1,4)=X(1,1)
       CMAT(2,4)=57.29578*X(2,1)/(1738.-X(1,1))
       CMAT(3,4)=X(3,1)
       CMAT(4,4)=X(4,1)
C   PUT THE APPROPIATE VALUES IN DISKFILES
       WRITE(5,4001) TIME, (CMAT(I,4),I=1,4)
4001   FORMAT (' TIME = ',F6.2,1P4G15.5)
       WRITE(1,200) CMAT(1,4)
       WRITE(2,200) CMAT(2,4)
       WRITE(3,200) CMAT(3,4)
       WRITE(4,200) CMAT(4,4)
       GO TO 22
       END
C    MATMULT IS ROUTINE TO MULTIPLY TWO MATRIX TOGETHER
C
       SUBROUTINE MATMUL (AMATX,BMATX,CMAT,SIZE)
       DOUBLE PRECISION CMAT
       INTEGER SIZE
       DIMENSION AMATX(4,4),BMATX(4,4)
       DIMENSION CMAT (4,4)
       DO 1 I=1,SIZE
       DO 2 J=1,SIZE
2      CMAT(I,J)=0.0
1      CONTINUE
C    MULTIPLY MATRICES
       DO 3 I=1,SIZE
       DO 4 K=1,SIZE
       DO 5 J=1,SIZE
       X=AMATX(I,J)*BMATX(J,K)
5      CMAT(I,K)=CMAT(I,K)+X
4      CONTINUE
3      CONTINUE
       RETURN
       END
       SUBROUTINE INVSUB (A,AI,RA,CA)
       DIMENSION A(4,4)
       DIMENSION IM(4,4),CMAT(4,4),AI(4,4)
       REAL IM
       INTEGER RA,CA
C   INITIALIZE ALL MATRICIES
       DO 1 I = 1,RA
       DO 2 J = 1,RA
       AI(I,J)=0.0
```

```
2       IM(I,J) = 0.0
1       CONTINUE
C
C   FORM THE IDENTITY MATRIX, IM
        DO 14 I=1,RA
        IM(I,I)=1.0
14      CONTINUE
C   INITIALIZE THE INVERSE MATRIX
C   START BY SETTING AI EQUAL TO A MATRIX
        DO 3 I=1,RA
        DO 4 J=1,CA
4       AI(I,J)=A(I,J)
3       CONTINUE
C   INVERT THE A MATRIX
C
C   FORWARD ROW REDUCTION STARTS HERE
        DO 50 K=1,CA
        LINE = K
9       FACTOR=AI(K,K)
        IF (FACTOR .EQ. 0.0) GO TO 2000
        LINE = K
        DO 71 I=1,RA
        AI(K,I)=AI(K,I)/FACTOR
        IM(K,I)=IM(K,I)/FACTOR
71      CONTINUE
        KPLUS1=K+1
        DO 20 I=KPLUS1 ,CA
        FACTOR=AI(I,K)
        DO 30 J=1,CA
        AI(I,J)=AI(I,J)-FACTOR*AI(K,J)
        IM(I,J)=IM(I,J)-FACTOR*IM(K,J)
30      CONTINUE
20      CONTINUE
50      CONTINUE
C REVERSE ROW REDUCTION STARTS HERE
        KSIZE=RA
89      KSM1=KSIZE-1
90      FACTOR=AI(KSM1,KSIZE)
        IF (FACTOR .EQ. 0.0) GO TO 330
        DO 310 J=1,RA
        AI(KSM1,J)=AI(KSM1,J)-FACTOR*AI(KSIZE,J)
        IM(KSM1,J)=IM(KSM1,J)-FACTOR*IM(KSIZE,J)
310     CONTINUE
330     KSM1=KSM1-1
        IF (KSM1 .GT. 0) GO TO 90
        KSIZE=KSIZE -1
        IF (KSIZE .GT. 1) GO TO 89
C INVERSION IS COMPLETE ASSIGN AI TO IM
        DO 60 I=1,RA
        DO 61 J=1,CA
61      AI(I,J)=IM(I,J)
60      CONTINUE
        GO TO 3002
2000    IF (K .EQ. CA) GO TO 3000
        LINE=LINE+1
        IF (LINE .GT. RA) GO TO 3000
C   INTERCHANGE LINES OF MATRIX
        LP1=LINE + 1
        DO 320 J=1,CA
        CMAT (K,J)=AI(K,J)
        CMAT (LP1,J)=IM(K,J)
        AI(K,J) = AI(LINE,J)
        IM(K,J) = IM(LINE,J)
        AI(LINE,J)=CMAT (K,J)
        IM(LINE,J)=CMAT (LP1,J)
```

```
320    CONTINUE
       GO TO 9
3000   WRITE (5,3001)
3001   FORMAT (' SINGULAR A MATRIX CAN NOT INVERT ')
3002   CONTINUE
C
       RETURN
       STOP
       END
```

VITA

Captain Allan T. Reed was born 18 September 1953 in Mountain Home, Idaho. He graduated in 1971 from North Miami High School in Denver, Indiana. He graduated with honors from Purdue University with a Bachelor of Science Degree in Aeronautical and Astronautical Engineering in 1975. Upon graduation he received a commission in the USAF as an ROTC Distinguished Graduate. Captain Reed graduated from pilot training from Craig AFB, Alabama in 1977 and was the recipient of the ATC Commander's Trophy. Initial operational experience was to Cannon AFB, New Mexico as an F-111D pilot. In 1980 he was selected to attend the F-111 Fighter Weapons School and upon graduation was selected as Distinguished Graduate. In 1982 Captain Reed served as an Instructor Pilot at the F-111 Fighter Weapons School at Mountain Home AFB, Idaho. In 1983 he was selected for the Joint Air Force Institute of Technology/Test Pilot School Program. The AFIT course work was completed in June 1984. Captain Reed graduated from the USAF Test Pilot School as a Distinguished Graduate in June 1985.

END

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